Cognitive Neuroscience II

Prof. Dr. Andreas Wendemuth

Lehrstuhl Kognitive Systeme

Institut für Elektronik, Signalverarbeitung und Kommunikationstechnik

Fakultät für Elektrotechnik und Informationstechnik
Otto-von-Guericke Universität Magdeburg

http://iesk.et.uni-magdeburg.de/ko/

People

- Prof. Dr.rer.nat. Andreas Wendemuth: 13 lectures

  Information Technology: Cognitive Systems
- Prof. Jochen Braun Ph.D.: 4 lectures

  Cognitive Neurobiology
- Dr.rer.nat. Sven Krüger: 2 lectures, 13 exercise classes

  Information Technology: Cognitive Systems

Contents

Literature (selected)

- Peter Dayan and L.F. Abbott: Theoretical Neuroscience Computational and Mathematical Modeling of Neural Systems, MIT Press, Cambridge 2001
- G.F. Luger et al., „Cognitive Science", Academic Press 1994 (Learning from an AI point of view)
- Stephen Andriole; Leonard Adelman: Cognitive systems engineering for user-computer interface design, prototyping, and evaluation

Network Models

- Introduction to biological network models
  (J. Braun)
- Dynamics /Associative Memory / stability
  (A. Wendemuth)
- Capacity /Coordinate Transforms
  (A. Wendemuth)
- Ex-/Inhibitory, Stochastic Networks
  (Sven Küger)

Dynamics & Coordinate Transforms

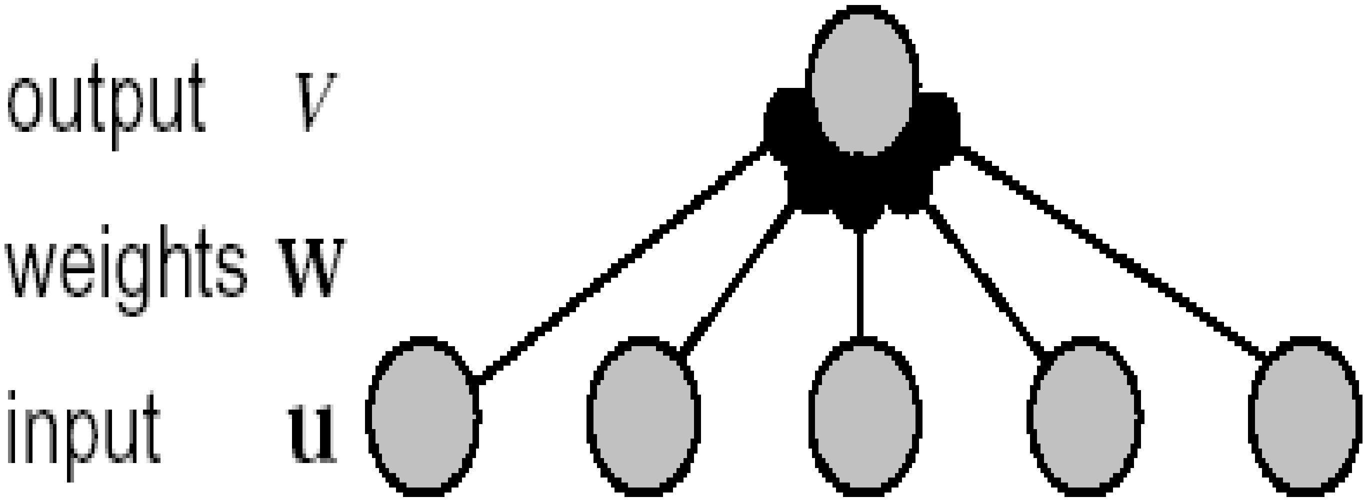
(A. Wendemuth, 13. April)

- Firing Rate
- Feedforward Networks
- Autoassociative Networks
- Stability
- Eigenvalue analysis
- Discretization

Firing Rate

- Activation function F(I): steady-state firing rate F as function of synaptic input current I
- Bounded from above, since excessively high rates are not observed: sometimes sigmoid or thresholded
- **u** = input firing rate vector
- Prediction by I= **w** * **u**

Network structure



Firing rate equation

- Membrane potential (resistance, capacitance) is low-pass filtered version of I_s:
- v = output firing rate

$$\tau \frac{dv}{dt} = -v + F(I(t)) = -v + F(\mathbf{w} * \mathbf{u})$$

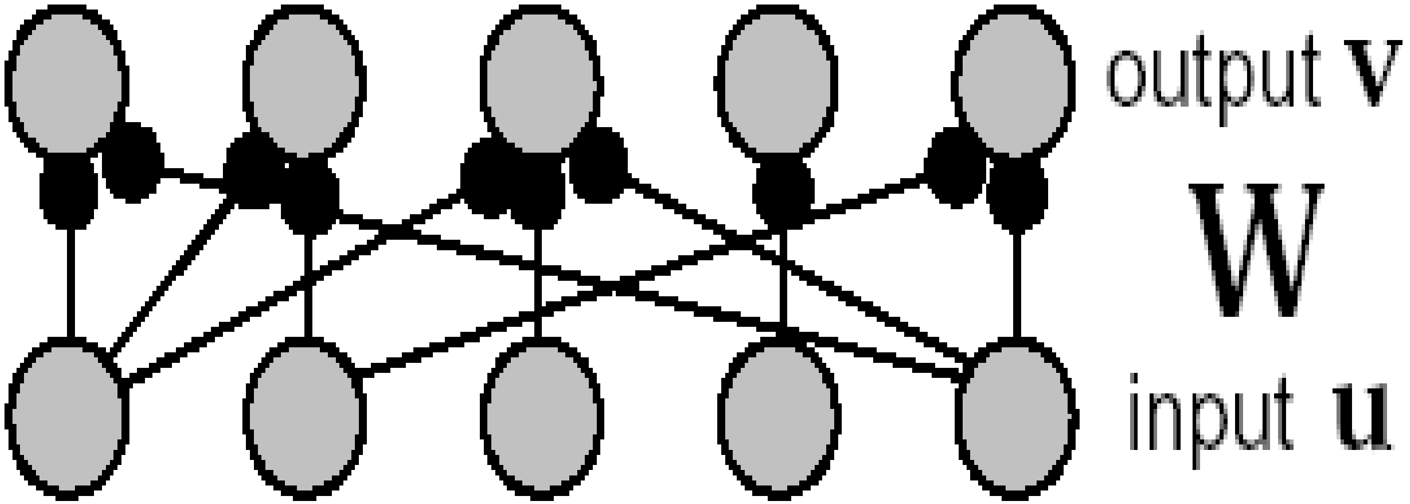
Feedforward Networks

- Input **u**, Output **v** (vectors!)

$$\tau \frac{d\mathbf{v}}{dt} = -\mathbf{v} + \mathbf{F}(\mathbf{I}(t)) = -\mathbf{v} + \mathbf{F}(\mathbf{W} * \mathbf{u})$$

- **W** is matrix of weights connecting neuron i in input layer to neuron j in output layer
- Activation $\mathbf{h} = \mathbf{W} * \mathbf{u}$

Structure of feedforward networks



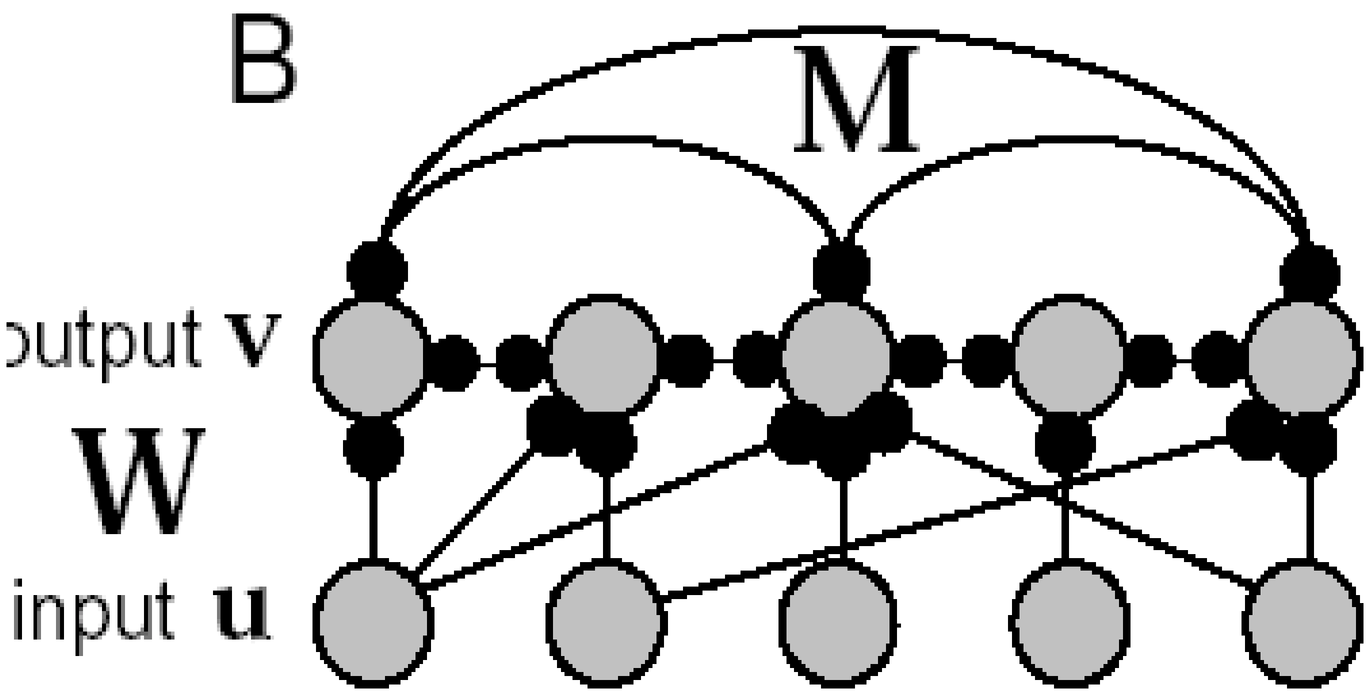
Recurrent Network

- Has weights **M** that couple to itself on same layer:

$$\tau \frac{d\mathbf{v}}{dt} = -\mathbf{v} + \mathbf{F}(\mathbf{h} + \mathbf{M}^*\mathbf{v})$$

- Dale‘s law: neurons either excite or inhibit, i.e. all signs are equal in one column of **M, W**

Structure of Recurrent networks



Autoassociative Networks

- Can be derived from recurrent networks: only couplings within the same layer, i.e.

$$\tau \frac{d\mathbf{v}}{dt} = -\mathbf{v} + \mathbf{F}(\mathbf{M}^*\mathbf{v})$$

1.Steady State Solution

- If no changes over time, steady state $\mathbf{v}_\infty$:

$$\mathbf{0} = -\mathbf{v}_\infty + \mathbf{F}(\mathbf{M}^*\mathbf{v}_\infty)$$

- Expansion of DEQ at $\mathbf{v} = \mathbf{v}_\infty + \Delta\mathbf{v}$ (derived on blackboard)

$$\tau \frac{d\Delta\mathbf{v}}{dt} = (-\mathbf{1} + diag(\frac{\partial F_j}{\partial x_j})_{[\mathbf{x} = \mathbf{M}^*\mathbf{v}_\infty]} * \mathbf{M}) * \Delta\mathbf{v}$$

Steady State Expansion

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$$\tau \frac{d\Delta\mathbf{v}}{dt} = \mathbf{A} * \Delta\mathbf{v}$$

with

$$\mathbf{A} = -\mathbf{1} + diag\left(\frac{\partial F_j}{\partial x_j}\right)_{[\mathbf{x} = \mathbf{M}*\mathbf{v}_\infty]} * \mathbf{M}$$

Stability: General Theorem

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$$\tau \frac{d\Delta \mathbf{v}}{dt} = \mathbf{A} * \Delta \mathbf{v}$$

is stable only if and only if all eigenvalues λ of **A** satisfy

$$\frac{1}{\tau}\lambda = \frac{1}{\tau} ev(\mathbf{A}) < 0$$

Eigenvalues λ of **A**

- Satisfy / are given by

$$\det(\mathbf{A} - \lambda \mathbf{1}) = 0$$

- and corresponding eigenvectors $\boldsymbol{\mu}$

$$(\mathbf{A} - \lambda_j \mathbf{1})\boldsymbol{\mu}_j = 0$$

Expansion in eigenvectors

- Expand for any vector **x**:

$$\mathbf{x} = \sum_j a_j \boldsymbol{\mu}_j$$

- Then

$$\mathbf{Ax} = \mathbf{A} \sum_j a_j \boldsymbol{\mu}_j = \sum_j a_j \mathbf{A} \boldsymbol{\mu}_j = \sum_j a_j \lambda_j \boldsymbol{\mu}_j$$

2. Back to DEQ: Stability

υ $$\tau \frac{d\Delta\mathbf{v}}{dt} = \mathbf{A} * \Delta\mathbf{v}$$

$\Delta\mathbf{v}$ Expanded: $$\Delta\mathbf{v} = \sum_j a_j \boldsymbol{\mu}_j$$

$$\tau \sum_j a_j \frac{d\boldsymbol{\mu}_j}{dt} = \sum_j a_j \lambda_j \boldsymbol{\mu}_j$$

Relaxation

- Since eigenvectors lin.indep., this must hold for each eigenvector

$$\tau \frac{d\boldsymbol{\mu}_j}{dt} = \lambda_j \boldsymbol{\mu}_j$$

- Resulting in

$$\boldsymbol{\mu}_j = \boldsymbol{\mu}_j(t=0) * \exp(\frac{\lambda_j}{\tau} t)$$

Condition

- Since $$\Delta \mathbf{v} = \sum_j a_j \boldsymbol{\mu}_j$$

  all $$\boldsymbol{\mu}_j = \boldsymbol{\mu}_j(t=0) * \exp(\frac{\lambda_j}{\tau} t)$$

  must relax to 0, which is the case if and only if

  $$\frac{\lambda_j}{\tau} < 0$$ Then the DEQ is locally (!) stable.

Example: steady state & stability

- Let $F_k(x) = \tanh(x)$, and card(M) = 2
- Then for j = 1,2:

$$\tau \frac{dv_j}{dt} = -v_j + \tanh(\mathbf{M^*v})_j$$

- Steady state

$$\mathbf{0} = \mathbf{g}(\mathbf{v}) = -\mathbf{v}_\infty + \mathbf{F}(\mathbf{M^*v}_\infty)$$

Steady state

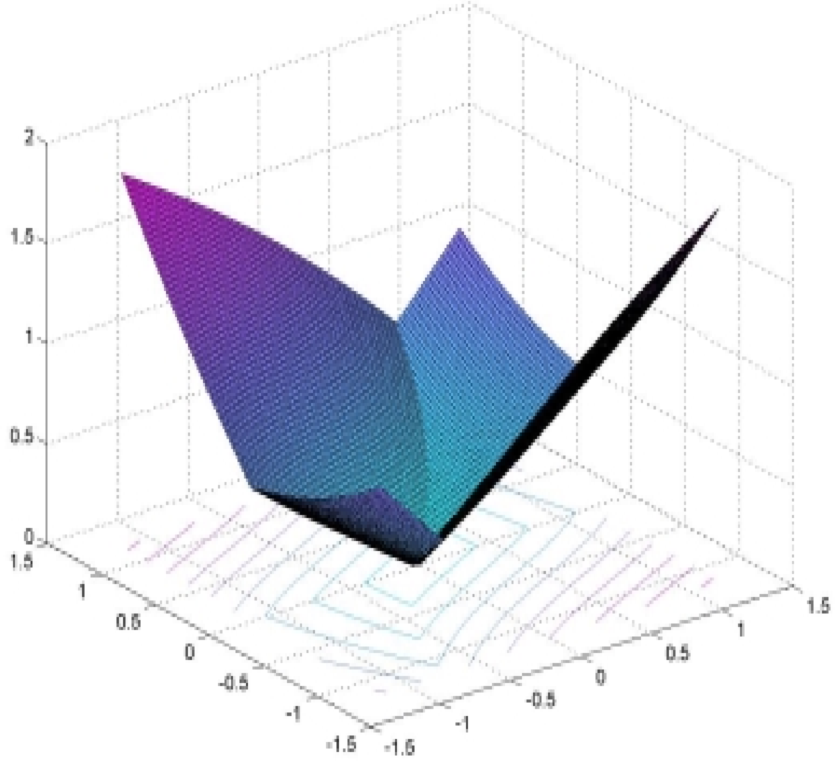
- Is only given if

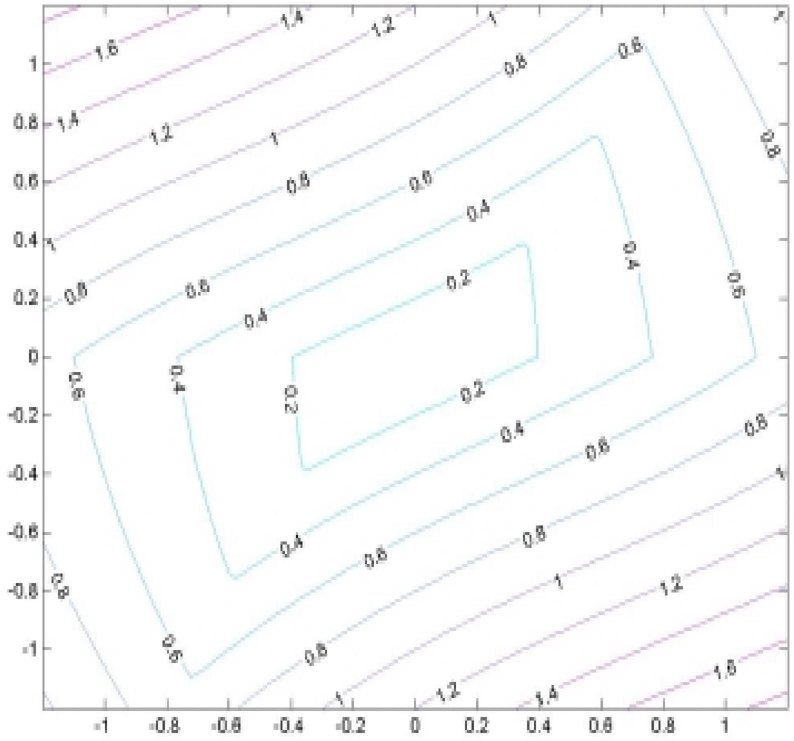
$$0 = |\mathbf{g(v)}| = \sum_j |-v_j + \tanh(\mathbf{M*v})_j|$$

- Example 1: $\mathbf{M} = \begin{pmatrix} 0.5 & 0.5 \\ 0 & 0.5 \end{pmatrix}$

$$|\mathbf{g(v)}| = |-v_1 + \tanh(0.5(v_1 + v_{2)}) | + | -v_2 + \tanh(0.5 v_2)|$$

Fixed Point Solution & Contour plot of |**g**(**v**)|





Stability analysis of Solution (0,0)

- Need

$$\mathbf{A} = -\mathbf{1} + diag(\frac{\partial F_j}{\partial x_j})_{[\mathbf{x}=\mathbf{M}*\mathbf{v}_\infty]} * \mathbf{M}$$

$$\mathbf{A} = -\mathbf{1} + \begin{pmatrix} 1/\cosh^2(x_1) & - \\ - & 1/\cosh^2(x_2) \end{pmatrix}_{[\mathbf{x}=\mathbf{0}]} * \begin{pmatrix} 0.5 & 0.5 \\ 0 & 0.5 \end{pmatrix}$$

$$\mathbf{A} = \begin{pmatrix} -0.5 & 0.5 \\ 0 & -0.5 \end{pmatrix}$$

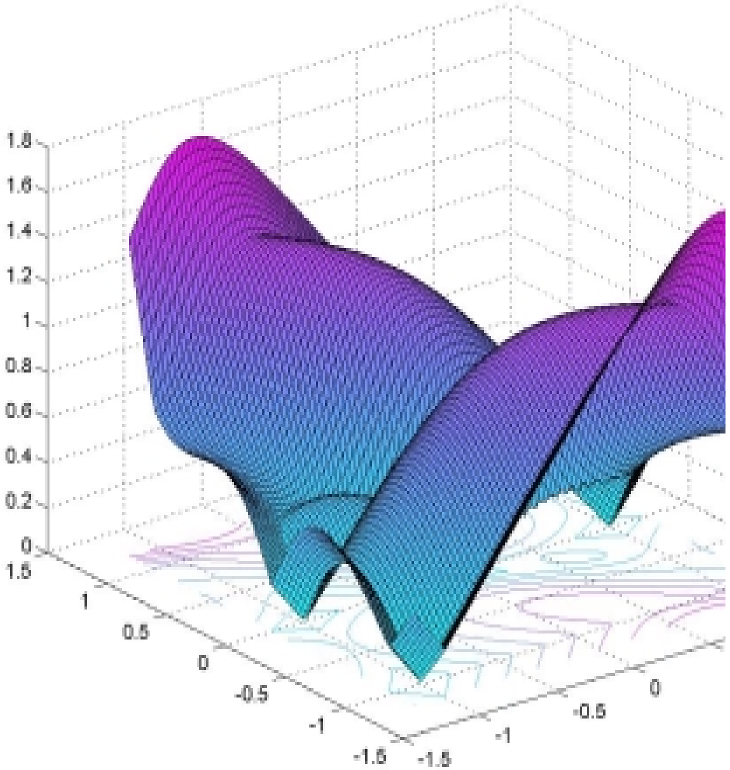
Eigenvalues

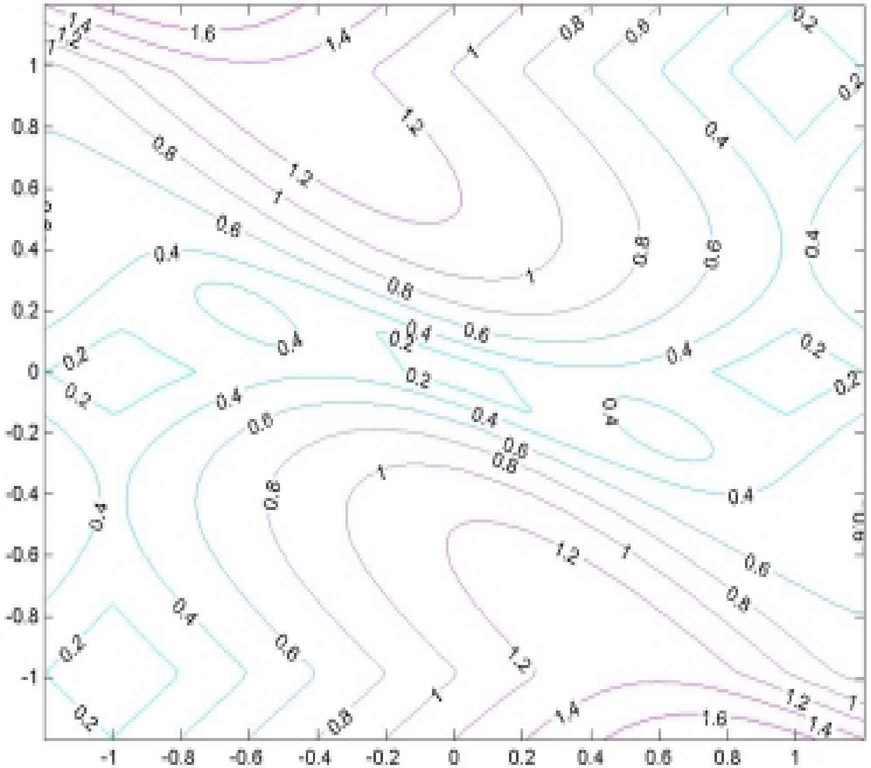
- Have

$$\det(\mathbf{A}-\lambda\mathbf{1}) = \det\begin{pmatrix} -0.5-\lambda & 0.5 \\ 0 & -0.5-\lambda \end{pmatrix} = (0.5+\lambda)^2 = 0$$

- Hence $\lambda = -0.5 < 0$: stable!

Example 2:
Same structure, $\mathbf{M} = \begin{pmatrix} 2.5 & 2.5 \\ 0 & 2.5 \end{pmatrix}$





-> 4 further solutions

- Ca. values:

- $(v_1, v_2) = (0.985, 0)$
- $(v_1, v_2) = (-0.985, 0)$
- $(v_1, v_2) = (1, 1)$
- $(v_1, v_2) = (-1, -1)$

Stability of Solutions v=(±0.985;0)

- Need

$$\mathbf{A} = -\mathbf{1} + diag\left(\frac{\partial F_j}{\partial x_j}\right)_{[\mathbf{x}=\mathbf{M}*\mathbf{v}_\infty]} * \mathbf{M}$$

$$\mathbf{A} = -\mathbf{1} + \begin{pmatrix} 1/\cosh^2(x_1) & - \\ - & 1/\cosh^2(x_2) \end{pmatrix}_{\left[\mathbf{x}=\begin{pmatrix} \pm 2.46 \\ 0 \end{pmatrix}\right]} * \begin{pmatrix} 2.5 & 2.5 \\ 0 & 2.5 \end{pmatrix}$$

$$\mathbf{A} = \begin{pmatrix} -0.93 & 0.07 \\ 0 & 1.5 \end{pmatrix}$$

Eigenvalues

- Have

$$\det(\mathbf{A}-\lambda\mathbf{1}) = \det\begin{pmatrix} -0.93-\lambda & 0.07 \\ 0 & 1.5-\lambda \end{pmatrix} = 0$$

- Hence $\lambda_1 = -0.93 < 0; \lambda_2 = 1.5 > 0$: instable!

Ex 1: Stability of Solutions v=± (1;1)

- Exercise 1:
- Perform the same analysis for these fixed points.

3. Discrete evolution in time

- The autoassociative DEG

$$\tau \frac{d\mathbf{v}}{dt} = -\mathbf{v} + \mathbf{F}(\mathbf{M}^*\mathbf{v})$$

in discrete time steps (superscripts)

$$\frac{\tau}{\Delta t}(\mathbf{v}^{t+1} - \mathbf{v}^{t}) = -\mathbf{v}^{t} + \mathbf{F}(\mathbf{M}^*\mathbf{v}^{t})$$

In particular, choosing $\Delta t = \tau$

$$\mathbf{v}^{t+1} = \mathbf{F}(\mathbf{M}^*\mathbf{v}^{t})$$

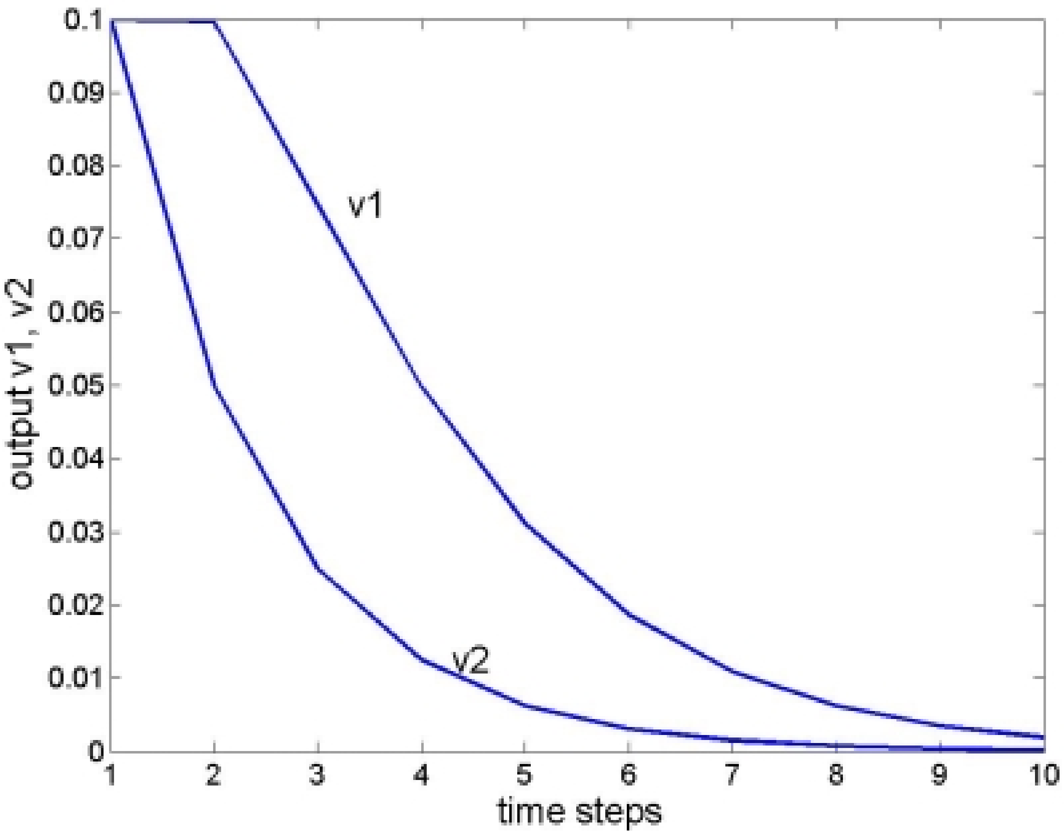
Example: discrete time steps

- This can be used for computer models.
- Our previous example 1: (j=1,2)

$$v_j^{t+1} = \tanh\left(\begin{pmatrix} 0.5 & 0.5 \\ 0 & 0.5 \end{pmatrix} * \mathbf{v}^t\right)_j$$

- If we start with **v** = (0.1;0.1),
  the time evolution looks like

Time evolution in example 1:



Further starting points

- Matlab example: live demo given.

Ex 2: Domain of stability

- Program this discrete evolution with a corresponding plot in Matlab.
- Start further „away“ from **v** = (0;0). Does the system become instable? / When does this happen / Why?
- Use $$\frac{\tau}{\Delta t}(\mathbf{v}^{t+1} - \mathbf{v}^{t}) = -\mathbf{v}^{t} + \mathbf{F}(\mathbf{M}{*}\mathbf{v}^{t})$$

  with other values $\frac{\tau}{\Delta t} \neq 1$. Discuss.

Ex 3: Discrete evolution

- The discrete version is:

$$v_j^{t+1} = \tanh\left(\begin{pmatrix} 2.5 & 2.5 \\ 0 & 2.5 \end{pmatrix} * \mathbf{v}^t \right)_j$$

- Exercise: Program this discrete evolution with a corresponding plot in Matlab. Initialize in the vicinity of the 5 found fixed points.

Resumé so far

- A simple autoassociative network with 2 neurons was considered,
  partially connected = 3 of 4 weights
- Depending on the values of the weights, this network can have 1 or 5 fixed points where the dynamics comes to a halt. I.e. at these points, the firing rates **v** are constant.
- The fixed point **v**=(0,0) (no firing) is stable.
- There are other fixed points **v**≠(0,0) which are unstable, i.e. we have seen that small deviations from this point will not vanish in time, but add up to large amounts.
- The behaviour can be simulated in discrete steps. The role of Δt and τ must be discussed.