Speech Signal Processing

Exercise 6

1. Hidden Markov Models

(a) The crazy soda machine
Given is a soda machine which gives one of three soft drinks when a button is pressed. Model the behaviour of the machine with a two state (s1, s2) HMM. The state transition matrix is given by

$$A = \begin{pmatrix} 0.7 & 0.3 \\ 0.5 & 0.5 \end{pmatrix} \tag{1}$$

The emission probabilities for the soft drinks are:

$$\begin{array}{ll} \text{cola}_{s1} = 0.6 & \text{cola}_{s2} = 0.1 \\ \text{tee}_{s1} = 0.1 & \text{tee}_{s2} = 0.7 \\ \text{pop}_{s1} = & \text{pop}_{s2} = \end{array}$$

i. What are the emission probabilities for pop in state $s1$ and $s2$?

ii. Give a formular for the emission b_{ijk} (with states s_i and s_j and the drink k). Assume that first a state transition will occur and then a drink will be released.

iii. The automation is in state $s1$. What is the probability that first a pop and then a tee is released (emitted)?

(b) Given are two HMMs with state transition probabilities A_1 and A_2:

$$A_1 = \begin{pmatrix} 0.2 & 0.8 & 0.0 \\ 0.0 & 0.6 & 0.4 \\ 0.0 & 0.0 & 1.0 \end{pmatrix} \quad A_2 = \begin{pmatrix} 0.6 & 0.4 & 0.0 \\ 0.0 & 0.8 & 0.2 \\ 0.0 & 0.0 & 1.0 \end{pmatrix}$$

Additionally the emitting probabilities are given as B_1 and B_2:

$$B_1 = \begin{pmatrix} 0.8 & 0.0 & 0.2 \\ 0.2 & 0.7 & 0.3 \\ 0.0 & 0.3 & 0.5 \end{pmatrix} \quad B_2 = \begin{pmatrix} 0.7 & 0.4 & 0.5 \\ 0.2 & 0.6 & 0.0 \\ 0.1 & 0.0 & 0.5 \end{pmatrix}$$

where the columns of B_1 and B_2 represent the states, and the rows stand for the symbols a, b, c. The start probabilities of both HMMs are:

$$\pi = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

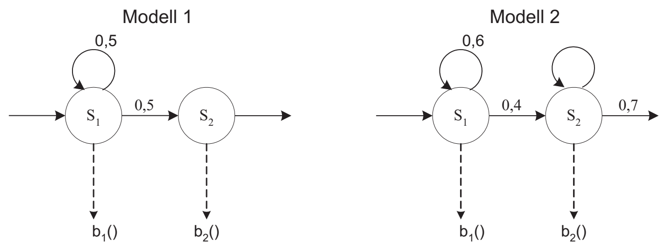
i. Sketch all possible paths through the two modells for 5 time steps (the last time step must be in the last state). Use the state transition matrices to constrain the number of possible paths through the models.

ii. Now use the emission probability matrices to further restrict the number of possible paths which can be traversed while producing the symbol sequence $O = \{aabcc\}$.

iii. Calculate the probability of producing $O = \{aabcc\}$ for both HMMs.

iv. Compute the Viterbi-probabilities and sketch the most probable path through the states.

(c) Estimation of HMM parameters
Given are the following two HMMs and the observation sequences



$O_1 = (o_1 = 1, o_2 = 2, o_3 = 4)$ and $O_2 = (o_1 = 3, o_2 = 3, o_3 = 3)$ which should be used to estimate the parameters of the first and second model respectively.

Sketch all allowed paths through the models given the observations.

(d) Use the Baum-Welch algorithm to estimate the emission probabilities b_1 and b_2 and the transition probability matrices.
The start probabilities for HMM 1 are:
$\mu_1 = 1$ and $\sigma_1 = 1$ and $\mu_2 = 2$ and $\sigma_2 = 1$
The start probabilities for HMM 2 are:
$\mu_1 = 1$ and $\sigma_1 = 1$ and $\mu_2 = 4$ and $\sigma_2 = 1$

(e) Which of the models did most probably produce the observation sequence $O_{Test}(o_1 = 2, o_2 = 3, o_3 = 4)$ (Use the forward algorithm)?

Anhang

Gaussian emission probabilities:

$$b_i(o_t) = \frac{1}{\sigma_i\sqrt{2\pi}} \cdot exp[-\frac{1}{2}(\frac{o_t - \mu_i}{\sigma_i})^2]$$

Computation of the alpha-matrix:

$$\alpha_j(t) = \left(\sum_{i=1}^{N} \alpha_i(t-1)a_{ij}\right) b_j(o_t) \quad \text{with} \quad 1 < j < N, 1 < t \leq T$$

Boundary condition: $\alpha_j(1) = a_{1j}b_j(o_1)$

Computation of the beta-matrix:

$$\beta_i(t) = \sum_{j=1}^{N} a_{ij}b_j(o_{t+1})\beta_j(t+1) \quad \text{with} \quad 1 < i < N, 1 \leq t < T$$

Boundary condition: $\beta_i(T) = 1$

Re-estimation of the transition probabilities $\hat{a}_{ij}$:

$$\hat{a}_{ij} = \frac{\sum_{t=1}^{T-1} \alpha_i(t) a_{ij} b_j(o_{t+1}) \beta_j(t+1)}{\sum_{t=1}^{T} \alpha_i(t) \beta_i(t)} \quad \text{with:} \quad 1 < i < N, 1 < j < N$$

with: $\hat{a}_{1j} = 1/P\alpha_j(1)\beta_j(1)$

$$\hat{a}_{iN} = \frac{\alpha_i(T)\beta_i(T)}{\sum_{t=1}^{T} 1/P\alpha_i(t)\beta_i(t)}$$

Re-estimation of the mean values $\hat{\mu}_j$:

$$\hat{\mu}_j = \frac{\sum_{t=1}^{T} \frac{1}{P} \alpha_j(t)\beta_j(t) o_t}{\sum_{t=1}^{T} \frac{1}{P} \alpha_j(t)\beta_j(t)} \quad \text{with:} \quad P = P(O|\lambda) = \alpha_N(T)$$