Lecture Speechprocessing

Exercise 4

1. Hidden Markov Modells

You can solve the following tasks using Matlab, but at least outline the approach!

(a) The crazy Vending Machine
Given a vending machine, which outputs one of three Beverages after pressing the button. Sketch a HMM representing this machine with two states (S1, S2) and the following transition matrix:

$$A = \begin{pmatrix} 0.7 & 0.3 \\ 0.5 & 0.5 \end{pmatrix}$$

The output probability for each beverage is:

$$\begin{array}{ll} \text{Cola}_{s1} = 0.6 & \text{Cola}_{s2} = 0.1 \\ \text{Tee}_{s1} = 0.1 & \text{Tee}_{s2} = 0.7 \\ \text{Soda}_{s1} = & \text{Soda}_{s2} = \end{array}$$

i. What are the output probabilities for soda in state $s1$ and $s2$?

ii. Enter the probability expression for output b_{ijk} (using the states s_i and s_j and teh beverage k and considering that first there is a transition and then an output.

iii. The machine is in state $s1$. How likely is first issued a soda and then a tea?

(b) Given two HMMs with the transition probabilities A_1 and A_2:

$$A_1 = \begin{pmatrix} 0.2 & 0.8 & 0.0 \\ 0.0 & 0.6 & 0.4 \\ 0.0 & 0.0 & 1.0 \end{pmatrix} \quad A_2 = \begin{pmatrix} 0.6 & 0.4 & 0.0 \\ 0.0 & 0.8 & 0.2 \\ 0.0 & 0.0 & 1.0 \end{pmatrix}$$

and the production probability matrices B_1 and B_2:

$$B_1 = \begin{pmatrix} 0.8 & 0.0 & 0.2 \\ 0.2 & 0.7 & 0.3 \\ 0.0 & 0.3 & 0.5 \end{pmatrix} \quad B_2 = \begin{pmatrix} 0.7 & 0.4 & 0.5 \\ 0.2 & 0.6 & 0.0 \\ 0.1 & 0.0 & 0.5 \end{pmatrix}$$

the columns of B_1 and B_2 represent the states, the rows the values a, b, c. The start probability for both models is:

$$\pi = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

i. Draw all possible paths for 5 time steps considering the transition probabilities for both models (the last time step must be in the final state).

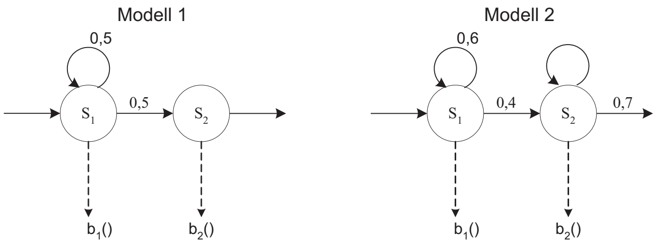
ii. We are now looking for all possible paths for the production of the string $O = \{aabcc\}$. Reduce the possible paths taking into account the Production probabilities.

iii. Calculate the total probabilities for both models.

iv. Calculate the Viterbi probabilities and illustrate the most likely path in the graph.

(c) Estimation of HMM parameters

Given the following two HMMs and an observation $O_1 = (o_1 =$



$1, o_2 = 2, o_3 = 4)$ to estimate the parameters of the first model and the observation sequence $O_2 = (o_1 = 3, o_2 = 3, o_3 = 3)$ to estimate the parameters of the second model.

Sketch all permissible ways of observations by the respective models.

(d) Appreciate using the Baum-Welch method, the production probabilities b_1 and b_2 and the transition probabilities.
The start parameters for model 1 are:
$\mu_1 = 1$ and $\sigma_1 = 1$ and $\mu_2 = 2$ and $\sigma_2 = 1$
The start parameters for model 2 are:
$\mu_1 = 1$ and $\sigma_1 = 1$ and $\mu_2 = 4$ and $\sigma_2 = 1$

(e) Which model has most likely produced the observation $O_{test}(o_1 = 2, o_2 = 3, O_3 = 4)$ (use the forward algorithm)?

Appendix

Gaussian distributed production probabilities:

$$b_i(o_t) = \frac{1}{\sigma_i\sqrt{2\pi}} \cdot exp[-\frac{1}{2}(\frac{o_t - \mu_i}{\sigma_i})^2]$$

Calculation of Alpha-matrix:

$$\alpha_j(t) = \left(\sum_{i=1}^{N} \alpha_i(t-1)a_{ij}\right) b_j(o_t) \quad \text{mit} \quad 1 < j < N, 1 < t \leq T$$

$$\text{Randbedingung: } \alpha_j(1) = a_{1j}b_j(o_1)$$

Calculation of Beta-Matrix:

$$\beta_i(t) = \sum_{j=1}^{N} a_{ij}b_j(o_{t+1})\beta_j(t+1) \quad \text{mit} \quad 1 < i < N, 1 \leq t < T$$

$$\text{Randbedingung: } \beta_i(T) = 1$$

Re-Estimation of transition probabilities $\hat{a}_{ij}$:

$$\hat{a}_{ij} = \frac{\sum_{t=1}^{T-1} \alpha_i(t) a_{ij} b_j(o_{t+1}) \beta_j(t+1)}{\sum_{t=1}^{T} \alpha_i(t) \beta_i(t)} \quad \text{mit:} \quad 1 < i < N, 1 < j < N$$

$$\text{mit: } \hat{a}_{1j} = 1/P \alpha_j(1) \beta_j(1)$$

$$\hat{a}_{iN} = \frac{\alpha_i(T) \beta_i(T)}{\sum_{t=1}^{T} 1/P \alpha_i(t) \beta_i(t)}$$

Re-Estimation of Means $\hat{\mu}_j$:

$$\hat{\mu}_j = \frac{\sum_{t=1}^{T} \frac{1}{P} \alpha_j(t) \beta_j(t) o_t}{\sum_{t=1}^{T} \frac{1}{P} \alpha_j(t) \beta_j(t)} \quad \text{mit:} \quad P = P(O|\lambda) = \alpha_N(T)$$