Lecture Speechprocessing

Exercise 3

1. Signal Estimation

   (a) Given is the signal $x(n)$, generated by an AR-Process.

   $$\begin{array}{lll} x(0) = 1,00 & x(1) = -0,75 & x(2) = -0,1042 \\ x(3) = 0,0781 & x(4) = 0,3859 & \end{array}$$

   Furthermore, the predictor-coefficients of different orders are given:

   $$\begin{array}{lllll} \text{1. Order:} & a_1 = -0,4 & & & \\ \text{2. Order:} & a_1 = -0,5556 & a_2 = -0,3889 & & \\ \text{4. Order:} & a_1 = -0,75 & a_2 = -0,6667 & a_3 = -0,5 & a_4 = 0,0 \end{array}$$

   (b) Calculate the function values for the function using the predictor coefficients of 1st, 2nd and 4th order $y(n)$, whereby $y(0) = 1$.

   (c) Sketch $x(n)$ and $y(n)$ (for each order) and calculate the errors $e(n)$ between $x(n)$ and $y(n)$.

   (d) What order did the original AR process probably have, from which the $x(n)$ arose?

2. Principal Component Analysis (PCA)

   (a) The following data values of the classes K_1 and K_2 are given:

   $$x_1 = \begin{pmatrix} -1 \\ 1 \end{pmatrix} x_2 = \begin{pmatrix} 2 \\ 2 \end{pmatrix} \quad y_1 = \begin{pmatrix} -2 \\ 0 \end{pmatrix} y_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

   (b) Sketch the distributions and calculate the class averages M_1, M_2 and the overall mean M_0.

   (c) Calculate the covariance matrix C and determine the eigenvalues.

   (d) Transform the data using the eigenvector of the largest eigenvalue in the one-dimensional space. Interpret you their results.

3. Linear Discriminant analysis (LDA)

   (a) Estimate the covariance matrices S_W and S_T using the previous task's data.

   (b) Calculate the LDA matrix and transform into the one-dimensional space.

   (c) Compare their results with the PCA.

4. Bayesian decision rule

   (a) What is the purpose of the Bayesian decision rule?

   (b) What is the formula for the Bayesian decision rule?

(c) A doctor has taken the following probabilities with respect to the disease his patient.

$$P(Bronchitis) = 0.05$$
$$P(Cough) = 0.25$$
$$P(Cough/Bronchitis) = 0.8$$

What do these numbers mean?

(d) What is the probability that the patient has bronchitis, if he coughs?

5. Maximum Likelihood Estimation

(a) What is the maximum likelihood Estimation?

(b) What requirements must be met?

(c) The following data of a distribution are given:

$$x_1 = 0.4 \quad x_2 = 1.2 \quad x_3 = 1.8 \quad x_4 = -1.0 \quad x_5 = 0.7 \quad x_6 = 0.2$$

The mean and variance of the data values should be estimated, assuming that the data are normally distributed.

(d) Another distribution has the following character:

$$f(t) = \begin{cases} \theta \cdot t^{\theta-1} & \text{für} t \in \{0,1\} \\ 0 & \text{sonst} \end{cases}$$

Estimate the unknown parameter $theta$ using the maximum likelihood approach.

6. Schätzen von Mischverteilungen mit dem Expectation-Maximization (EM) Algorithmus

You can solve the following task with Matlab, but should at least outline the approach.

(a) What is a Mixture distribution?

(b) Calculate the means μ_1 and μ_2 of a mixture distribution. Use the Kullback-Leibler distance and the following start values $\mu_1 = -12$, $\sigma_1 = 3.5$ as well as $\mu_2 = 11$, $\sigma_2 = 2.6$.
Given observations:

$$\begin{array}{llll} x_1 = -3.6 & x_2 = -3.95 & x_3 = -5.15 & x_4 = -2.55 \\ x_5 = 4.8 & x_6 = 5.35 & x_7 = 7.00 & x_8 = 6.05 \end{array}$$

The Kullback-Leibler distance is given by:

$$Q(\hat{\lambda}, \lambda) = -\sum_{k=1}^{K} \left(\sum_{i=1}^{N} y_{i,k} \left(ln\sqrt{2\pi} + ln\sigma_k + \frac{(x_i - \mu_k)^2}{2\sigma_k^2} \right) \right)$$

with

$$y_{i,k} = \frac{p_k P(x_i|\mu_k, \sigma_k)}{\sum_{\lambda=1}^{K} p_\lambda P(x_i|\mu_\lambda, \sigma_\lambda)}$$

and $p_k = 0.5 \forall k$. The probabilities $P(x_i|\mu_k, \sigma_k)$ are given by a Gaussian distribution:

$$P(x_i|\mu_k, \sigma_k) = \frac{1}{\sqrt{2\pi}\sigma_k} \quad e^{-\frac{(x_i-\mu_k)^2}{2\sigma_k^2}}$$

Appendix

PCA

The covariance matrix C is obtained for a centered distribution of Vectors bfx to

$$C = \frac{1}{N} \sum_{i=1}^{N} \mathbf{x}_i \cdot \mathbf{x}_i^T$$

LDA

$$S_W = \frac{1}{N} \sum_{j=1}^{J} N_j \cdot W_j \quad \text{mit} \quad W_j = \frac{1}{N_j} \sum_{i=1}^{N_j} (x_{i,j} - m_j)\,(x_{i,j} - m_j)^T$$

$$S_B = \sum_{j=1}^{J} \frac{N_j}{N} (m_j - m_0)\,(m_j - m_0)^T$$

$$S_T = \frac{1}{N} \sum_{i=1}^{N} (x_i - m_0)\,(x_i - m_0)^T$$

Determination of the LDA-matrix Θ: $S_W^{-1} S_T \Theta = \Theta \Lambda$
Transformation using the LDA matrix Θ: $Y = \Theta^T X$

Misc:

1. Eigenvalues and Eigenvectors
   For all eigenvalues $lambda$ of the matrix A applies:

   $$det(A - \lambda E_n) = 0$$

   If $lambda$ is an eigenvalue of the matrix A, then the associated eigenvector bfx defined by:

   $$(A - \lambda E_n)\mathbf{x} = 0$$

2. Inverse Matrix
   The inverse of a square matrix A is described by:

   $$A \cdot A^{-1} = E$$