Band Pass Signal Representation

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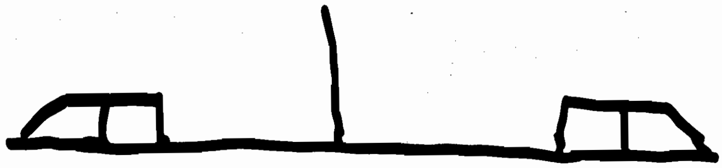
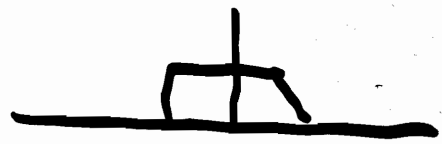


$s(t)$

Lowpass Equivalent

Low Pass Equivalent



Reasons to modulate

Reasons to modulate

* Antenna dimension
* Channel Characteristics
* Multiplexing
* Receiver Processing
* Equipment Cost

$s(t)$ B.P. Signal

$s_l(t)$ L.P. Eq. Sig

Pre Envelope od s(t)

$s_+(t)$ pre envelope of $s(t)$

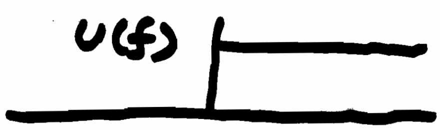
B.P. Complex

to keep same energy

$$S_+(f) = 2U(f)S(f)$$

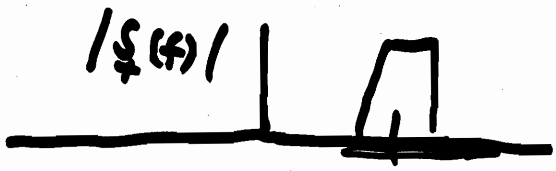
Unit Step fn

Unit step function



$$s_+(t) = \int_{-\infty}^{\infty} S_+(f) e^{-j2\pi ft} dt$$

$\therefore$ IFT of $S(f)$



$$s_+(t) = F^{-1}[2U(f)] * F^{-1}[S(f)]$$

IFT of $2U(f)$

IFT of 2U(f)

$$F^{-1}[2U(f)] = \delta(t) + \frac{j}{\pi t}$$

$$s_+(t) = F^{-1}[2U(f)] * s(t)$$

Hilbert Transform

$$\frac{1}{\pi t} * s(t) = \hat{s}(t)$$

$$s_+(t) = \left(\delta(t) + \frac{j}{\pi t}\right) * s(t)$$

$$= s(t) + \frac{j}{\pi t} * s(t)$$

$$= s(t) + \hat{s}(t)$$

$$\hat{s}(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{s(\tau)}{t-\tau} d\tau$$

$$s(t) \rightarrow \boxed{\frac{1}{\pi t}} \rightarrow \hat{s}(t)$$

Hilbert Transform

90 phase shifter

Hilbert Transform — 90° phase shifter

f	ph shift
+ve	-90°
−ve	$+90^\circ$

$$H(f) = \int_{-\infty}^{\infty} \frac{1}{\pi t} e^{-j\pi f t}\, dt$$

$$= \begin{cases} -j \; ; & f > 0 \\ 0 \; ; & f = 0 \\ +j \; ; & f < 0 \end{cases}$$

Magnitude

Magnitude

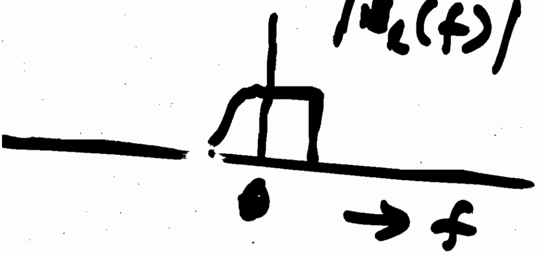
$$|H(f)| = 1 \quad \text{except } f = 0$$

Phase

Phase

$$\angle H(f) = \begin{cases} -\pi/2 & \text{for } f > 0 \\ \pi/2 & \text{for } f < 0 \end{cases}$$

$|S_l(f)|$



$s_l(t)$ L.P. Eq. of B.P. sig $s(t)$

$$S_l(f) = S_+(f + f_c)$$

$$s_l(t) = s_+(t)\, e^{-j2\pi f_c t}$$

but

$$s_+(t) = s(t) + \hat{s}(t)$$

but s+(t)=s(t)+ s`(t)

$$j\hat{s}(t) = (-j \sin 2\pi f_c t)$$

$$\therefore\; s_l(t) = [s(t) + \hat{s}(t)]\, e^{-j2\pi f_c t}$$

Low Pass Complex Signal (B. Band Signal)

let $s_l(t)$ has real & imaginary components

$$s_l(t) = x(t) + j\,y(t)$$

$$[s(t) + j\,\hat{s}(t)] = [x(t) + j\,y(t)]\, e^{j2\pi f_c t}$$

$$x(t) = s(t)\cos(2\pi f_c t) + \hat{s}(t)\sin 2\pi f_c t$$

$$y(t) = -s(t)\sin(2\pi f_c t) + \hat{s}(t)\cos 2\pi f_c t$$

$s(t) \xrightarrow[\text{Down Conversion}]{} s_l(t)$

B.P. Signal — L.P. Eq Complex Signal

B.P. Signal-------Down Conversion--------L.P. Eq. Complex Signal

L.P. Signal-------Up Conversion----------B.P.Signal

$s_l(t) \xrightarrow[\text{Up Conversion}]{} s(t)$

L.P. Sig — B.P. signal

L.P. to B.P.

L.P. to B.P.

$$s_l(t) = x(t) + j\,y(t)$$

$$s(t) = x(t)\cos 2\pi f_c t - y(t)\sin 2\pi f_c t$$

$$\hat{s}(t) = x(t)\sin 2\pi f_c t + y(t)\cos 2\pi f_c t$$

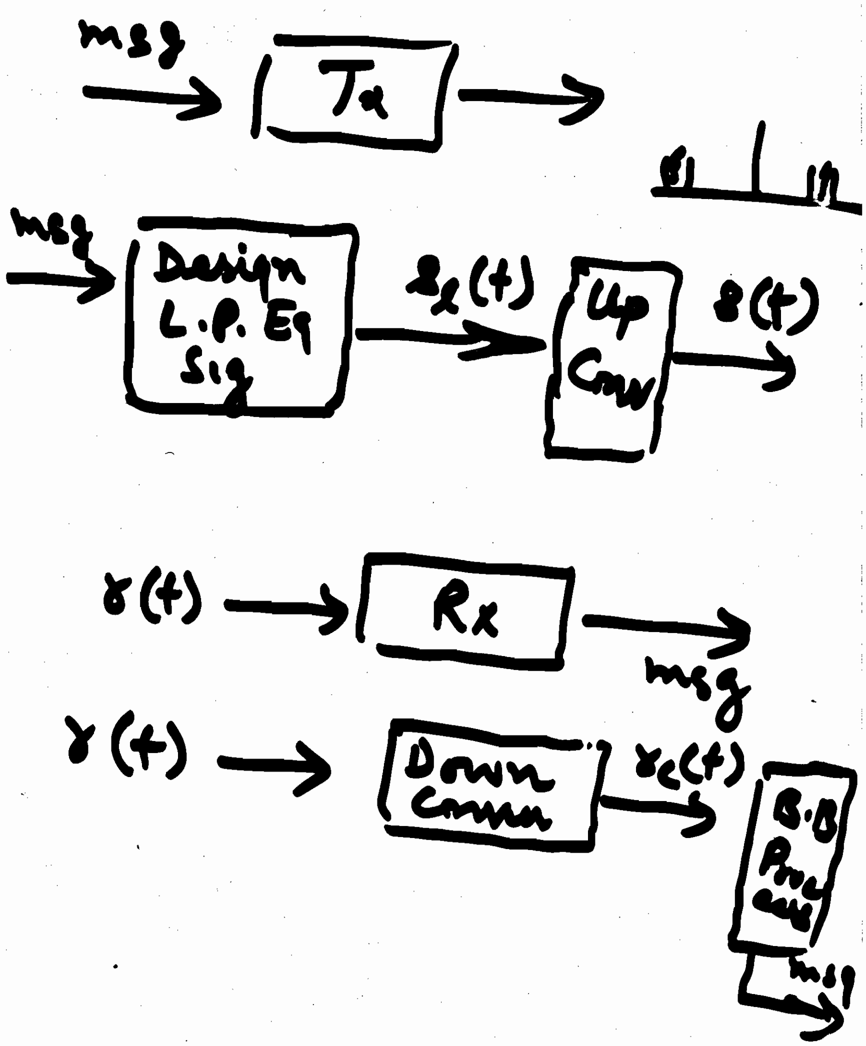
$\therefore s_l(t)$

$$s_l(t)\, e^{j2\pi f_c t} = s(t) + j\,\hat{s}(t)$$

$$\therefore S_l(f) = \underbrace{S_+(f)\, e^{-j2\pi f_c t}}_{[s(t)+j\,\hat{s}(t)]}$$

$\therefore$ r(t) processing complex costly in H.F.

So bring down to L.P. Eq & process in B.Band



r(t) processing complex
Costly in H.F.
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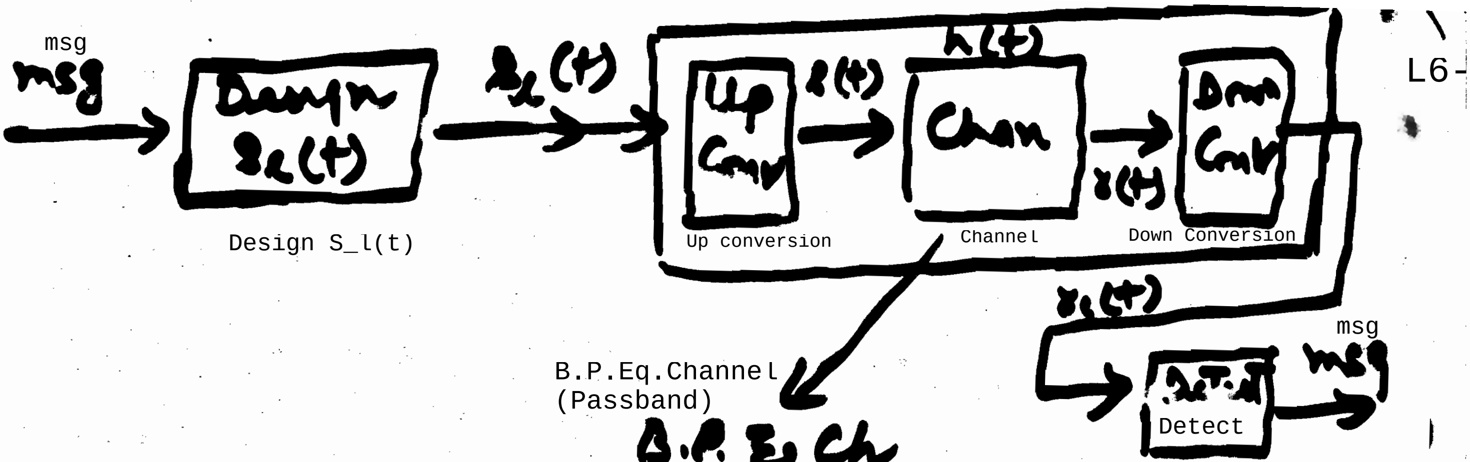
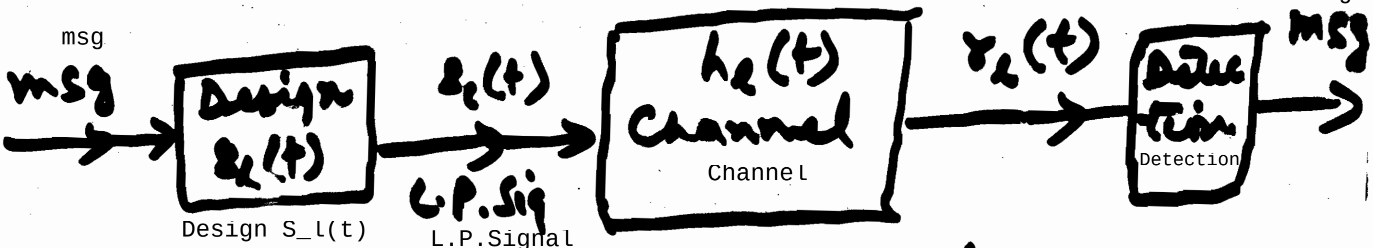


B.P.Eq.Channel
(Passband)

B.P. Eq Ch
(Passband)

$\Downarrow$

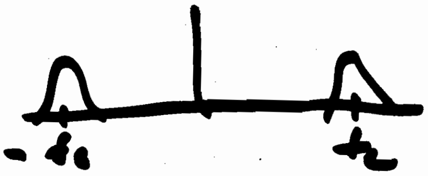


L.P.Eq.Channel
(Baseband)

L.P. Channel
(Baseband)

Complex Channel
Complex Signal

Complex Channel
Complex Signal

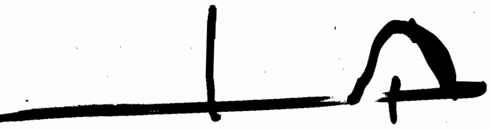


B.P.Signal

B.P. Signal

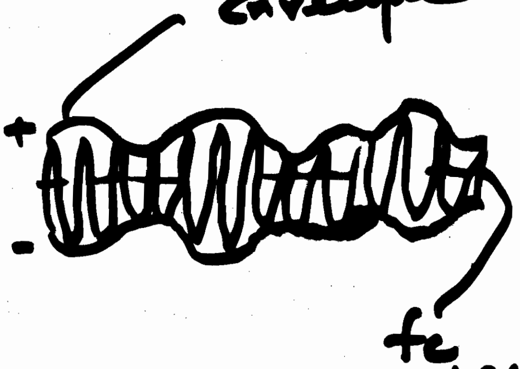
Suppress by Unit Impulse
to get one sided signal

Suppress by
Unit Impulse
to get one sided
Sig



Envelope

Envelope



If $s_l(t) = a(t)\, e^{j\theta(t)}$

Shift the spectrum
to get L.P.Eq.Sig

Shift the spectrum
to get L.P. Eq

$a(t) = \sqrt{x^2(t) + y^2(t)}$

$\theta(t) = \tan^{-1}\left(\frac{y(t)}{x(t)}\right)$

$s(t) = \mathrm{Re}\left[s_l(t)\, e^{j2\pi f_c t}\right]$

Envelope Phase

Envelope Phase

$s(t) = a(t) \cos(2\pi f_c t + \phi(t))$

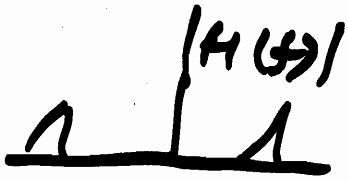
Energy of a B.P.Signal

Energy of a B.P. Signal

$$E = \int_{-\infty}^{\infty} s^2(t)\,dt \approx \frac{1}{2}\int_{-\infty}^{\infty} \left| s_l(t) \right|^2 dt$$

I.R. of a B.P. System H(f)

I.R. of a B.P. System $H(f)$



real, bandpass

$h(t) \rightarrow$ real, band pass

$$H(f) = H^*(-f)$$

conjugate shift

Conjugate Shift

$$H_+(f) = \begin{cases} H(f) & \text{if } f > 0 \\ 0 & \text{if } f < 0 \end{cases}$$

$$H_+(f) = U(f)\,H(f)$$

$$H_l(f) = H_+(f + f_c)$$

$$H_l(f - f_c) = \begin{cases} H(f) & ;\ f > 0 \\ 0 & ;\ f < 0 \end{cases}$$

$$H(f) = H_l(f - f_c) + H_l^*(-f - f_c)$$

$$h(t) = h_l(t)\,e^{j2\pi f_c t} + h_l^*\,e^{-j2\pi f_c t}$$

$$= 2\,\mathrm{Re}\left[h_l(t)\,e^{j2\pi f_c t}\right]$$

$$s(t) \longrightarrow \boxed{h(t)} \longrightarrow r(t)$$

$\downarrow$ LPE (LPEq) $\qquad\qquad$ $\downarrow$ LPE (LPEq)

$$s_l(t) \longrightarrow \boxed{h_l(t)} \longrightarrow r_l(t)$$

$$r(t) = \mathrm{Re}\left[r_l(t)\, e^{j2\pi f_c t}\right]$$

$$s(t) = \mathrm{Re}\left[s_l(t)\, e^{j2\pi f_c t}\right]$$

$$h(t) = 2\,\mathrm{Re}\left[h_l(t)\, e^{j2\pi f_c t}\right]$$

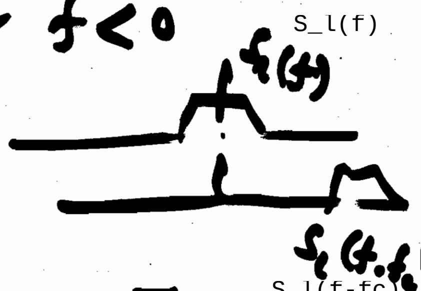
$$R(f) = S(f)\,H(f)$$

$$= \frac{1}{2}\left[S_l(f-f_c) + S_l^*(-f-f_c)\right]$$

$$= \left[H_l(f-f_c) + H_l^*(f-f_c)\right]$$

$s_l(t), h_l(t) \rightarrow$ narrowband signals

So $S_l(f-f_c) = 0$ for $f<0$

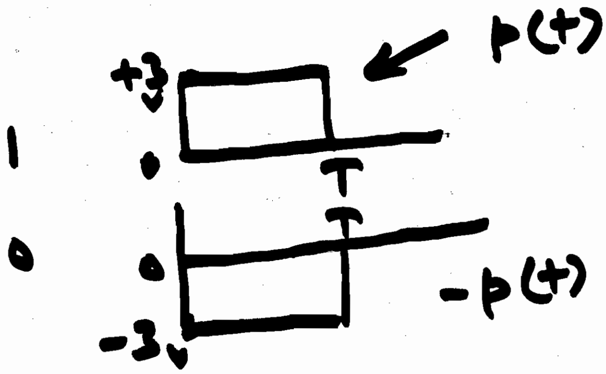


$$R(f) = \frac{1}{2}\left[S_l(f-f_c)\,H_l(f-f_c) + S_l^*(-f-f_c)\,H_l^*(-f-f_c)\right]$$

$$R(f) = \frac{1}{2}\left[R_l(f-f_c) + R_l^*(-f-f_c)\right]$$

$$R_l(f) = S_l(f)\,H_l(f)$$

$$\begin{tikzcd}
s(t) \arrow[r] \arrow[d, "\text{Down Convert}"', "\text{L.P. Eq}"] & \boxed{h(t)} \arrow[r] & r(t) \\
s_l(t) \arrow[r] & \boxed{h_l(t)} \arrow[r] & r_l(t) \arrow[u, "\text{Up Convert}"']
\end{tikzcd}$$

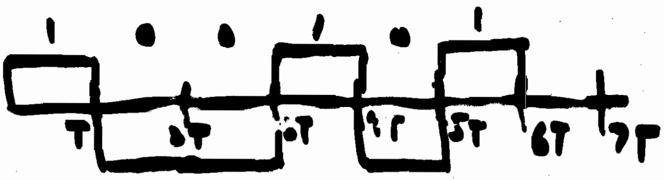


Mapping

$1 \longrightarrow +3V$

$0 \longrightarrow -3V$

1 0 0 1 0 1



1 1 0 0 1 0 1

$b_0 b_1 b_2 b_3 \ldots\ldots$

$a_0 a_1 a_2 a_3$

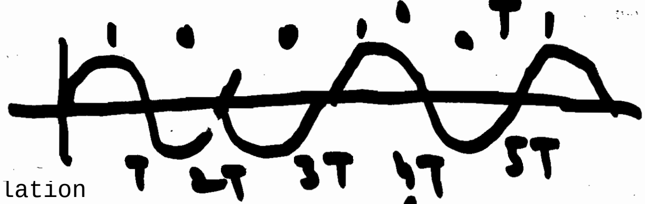
$a_0 p(t)$

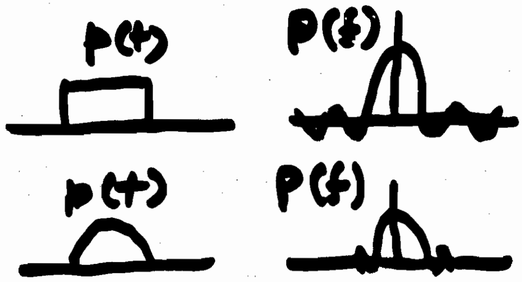
$a_1 p(t-T)$

$a_2 p(t-2T$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k \, p(t-kT)$$

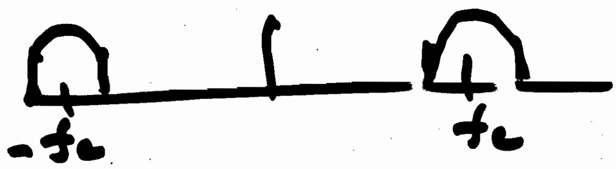
If $p(t)$ is  or 





Passband Modulation

If $p(t)$ is pass band



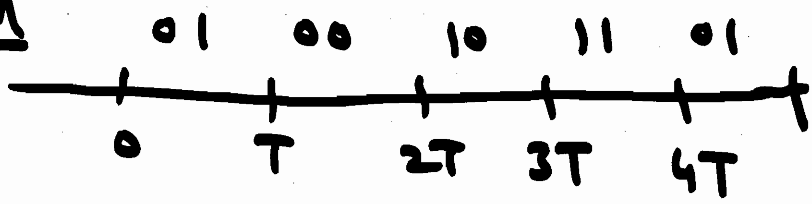
$p'(t) = p(t)\sin(2\pi f_c t)$

$= \frac{1}{2j} p(t)\left(e^{j2\pi f_c t} - e^{-j2\pi f_c t}\right)$

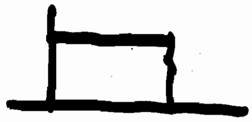
$\sum a_k \, p'(t-kT)$

$b(t) = \sum_{k=-\infty}^{\infty} a_k \, p(t-kT)$

4-PAM



00 ⟶ 1 ⟶ $p(t)$
01 ⟶ 2 ⟶ $2p(t)$
10 ⟶ 3 ⟶ $3p(t)$
11 ⟶ 4 ⟶ $4p(t)$



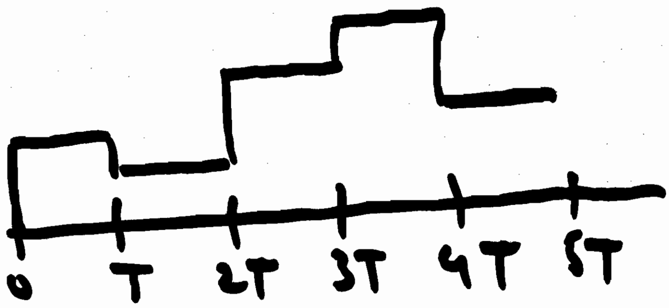
Advantage: More bits/s

Ad :- more bit/s

disadvantage: More Power

disAd :- more power

Avg power



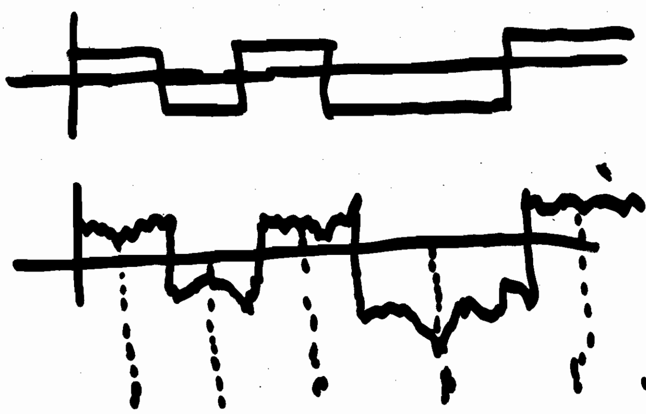
If we increase the levels the noise probability increases

If we increase the levels the noise probability increases

M-PAM

$M = 2^b$

b bits $= 2^b$ levels



AWGN(-inf --- inf)

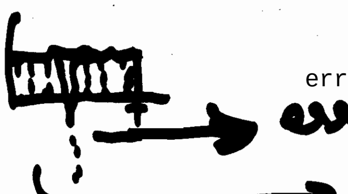
AWGN $(-\infty - \infty)$

Noise afect

Noise Effect

Sampling intant

Sampling Instant



error

error

Integrate to demodulate

integrate to demodulate

$$\int_0^T y(t)\,dt$$

Demodulation

[redacted] Demodulation

$$x(t) \longrightarrow a_k \longrightarrow b_k$$

$$\hat{a}_k = \int_{kT}^{(k+1)T} x(t)\,dt$$

kT $(k+1)T$

$$= \int_{-\infty}^{\infty} x(t)\, p(t-kT)\,dt$$

$p(t)$ 0 T

$$\int_{-\infty}^{\infty} x(t)\, p(T-(k+1)T+t)\,dt$$

0 kT $(k+1)T$

at time

$$p(T-t) * x(t) \Big|_{t=(k+1)T}$$

$$p(T-t) \rightarrow y(t)$$

$$\int y((k+1)T-t)\, x(t)\,dt$$

$$x(t) \longrightarrow \boxed{h(t) = p(T-t)} \longrightarrow \; mT \; \longrightarrow \hat{a}_k$$

$\hat{a}_0, \hat{a}_1, \ldots$

filtering filtering

sampling sampling

demapping demapping