Digital Transmission Through Bandlimited Channels

Chapter 6

(reference notes)

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Digital Communication Systems
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6.1 Preview

spectral characteristics of PAM signals.

the characterization of bandlimited channels

signal waveforms design

equalizers that compensate for distortion caused by bandlimited channels

channel distortion results in intersymbol interference (ISI),

causes errors in signal

reduces the error rate in the demodulated data sequence.

6.2 The Power Spectrum of a Digital PAM Signal

A digital PAM signal at the input to a communication channel is generally represented as

$$v(t) = \sum_{n=-\infty}^{\infty} a_n g(t - nT)$$

where $\{a_n\}$ is the sequence of amplitudes corresponding to the information symbols from the source, $g(t)$ is a pulse waveform, and T is the reciprocal of the symbol rate.

T is also called the *symbol interval*.

Each element of the sequence $\{a_n\}$ is selected from one of the possible amplitude values, which are

$$A_m = (2m - M + 1)d, \quad m = 0, 1, \ldots, M - 1$$

where d is a scale factor that determines the Euclidean distance between any pair of signal amplitudes ($2d$ is the Euclidean distance between any adjacent signal amplitude levels).

Since the information sequence is a random sequence, the sequence $\{a_n\}$ of amplitudes corresponding to the information symbols from the source is also random. Consequently, the PAM signal $v(t)$ is a sample function of a random process $V(t)$

To determine the spectral characteristics of the random process $V(t)$, we must evaluate the power spectrum.

First, we note that the mean value of $V(t)$ is

$$E[V(t)] = \sum_{n=-\infty}^{\infty} E(a_n) g(t - nT)$$

By selecting the signal amplitudes to be symmetric about zero equally probable, $E(a_n) = 0$, and hence $E[V(t)] = 0$.

The autocorrelation function of $V(t)$ is $R_v(t + \tau; t) = E[V(t)V(t + \tau)]$

It is shown in many standard texts on digital communications that the autocorrelation function is a periodic function in the variable t with period T. Random processes that have a periodic mean value and a periodic autocorrelation function are called *periodically stationary*, or *cyclostationary*. The time variable t can be eliminated by averaging $R_v(t + \tau; t)$ over a single period, i.e.,

$$\bar{R}_v(\tau) = \frac{1}{T} \int_{-T/2}^{T/2} R_v(t + \tau; t)\, dt$$

where $R_a(m) = E(a_n a_{n+m})$ is the autocorrelation of the sequence $\{a_n$ and $R_g(\tau)$ is defined as

$$R_g(\tau) = \int_{-\infty}^{\infty} g(t) g(t + \tau)\, dt$$

The power spectrum of $V(t)$ is simply the Fourier transform of the average autocorrelation function $\bar{R}_v(\tau)$, i.e.,

$$\mathcal{S}_v(f) = \int_{-\infty}^{\infty} \bar{R}_v(\tau) e^{-j2\pi f \tau}\, d\tau = \frac{1}{T} \mathcal{S}_a(f) |G(f)|^2$$

where $\mathcal{S}_a(f)$ is the power spectrum of amplitude sequence $\{a_n\}$ and $G(f)$ is the Fourier transform of the pulse $g(t)$.

$\mathcal{S}_a(f)$ is defined as

$$\mathcal{S}_a(f) = \sum_{m=-\infty}^{\infty} R_a(m) e^{-j2\pi f m T}$$

the power spectrum of the PAM signal is a function of the power spectrum of the information symbols $\{a_n\}$ and the spectrum of the pulse $g(t)$. In the special case where the sequence $\{a_n\}$ is uncorrelated, i.e.,

$$R_a(m) = \begin{cases} \sigma_a^2, & m = 0 \\ 0, & m \neq 0 \end{cases}$$

where $\sigma_a^2 = E(a_n^2)$, it follows that $\mathcal{S}_a(f) = \sigma_a^2$ for all f and $\mathcal{S}_v(f) = \frac{\sigma_a^2}{T} |G(f)|^2$

In this case, the power spectrum of $V(t)$ is dependent entirely on the spectral characteristics of the pulse $g(t)$.

6.3 Characterization of Bandlimited Channels and Channel Distortion

Many communication channels, including telephone channels and some radio channels, may be generally characterized as bandlimited linear filters. Consequently, such channels are described by their frequency response $C(f)$, which may be expressed as

$$C(f) = A(f)e^{j\theta(f)} \tag{6.3.1}$$

where $A(f)$ is called the *amplitude response* and $\theta(f)$ is called the *phase response*. Another characteristic that is sometimes used in place of the phase response is the *envelope delay*, or *group delay*, which is defined as

$$\tau(f) = -\frac{1}{2\pi}\frac{d\theta(f)}{df} \tag{6.3.2}$$

A channel is said to be *nondistorting*, or *ideal* if, within the bandwidth W occupied by the transmitted signal, $A(f)$ = constant and $\theta(f)$ is a linear function of frequency (or the envelope delay $\tau(f)$ = constant). On the other hand, if $A(f)$ and $\tau(f)$ are not constant within the bandwidth occupied by the transmitted signal, the channel distorts the signal. If $A(f)$ is not constant, the distortion is called *amplitude distortion*, and if $\tau(f)$ is not constant, the distortion on the transmitted signal is called *delay distortion*.

As a result of the amplitude and delay distortion caused by the nonideal channel frequency response characteristic $C(f)$, a succession of pulses transmitted through the channel at rates comparable to the bandwidth W are smeared to the point that they are no longer distinguishable as well-defined pulses at the receiving terminal. Instead, they overlap, so we have intersymbol interference. As an example of the effect of delay distortion on a transmitted pulse, Figure 6.4(a) illustrates a bandlimited pulse having zeros periodically spaced in time at points labeled $\pm T$, $\pm 2T$, etc. When the information is conveyed by the pulse amplitude, as in PAM, then one can transmit a sequence of pulses, each of which has a peak at the periodic zeros of the other pulses. However, transmission of the pulse through a channel modeled as having a linear envelope delay characteristic $\tau(f)$ [quadratic phase $\theta(f)$] results in the received pulse shown in Figure 6.4(b) having zero crossings that are no longer periodically spaced. Consequently, a sequence of successive pulses would be smeared into one another, and the peaks of the pulses would no longer be distinguishable. Thus the channel delay distortion results in intersymbol interference. As will be discussed in this chapter, it is possible to compensate for the nonideal frequency response characteristic of the channel by use of a filter or equalizer at the demodulator. Figure 6.4(c) illustrates the output of a linear equalizer that compensates for the linear distortion in the channel.

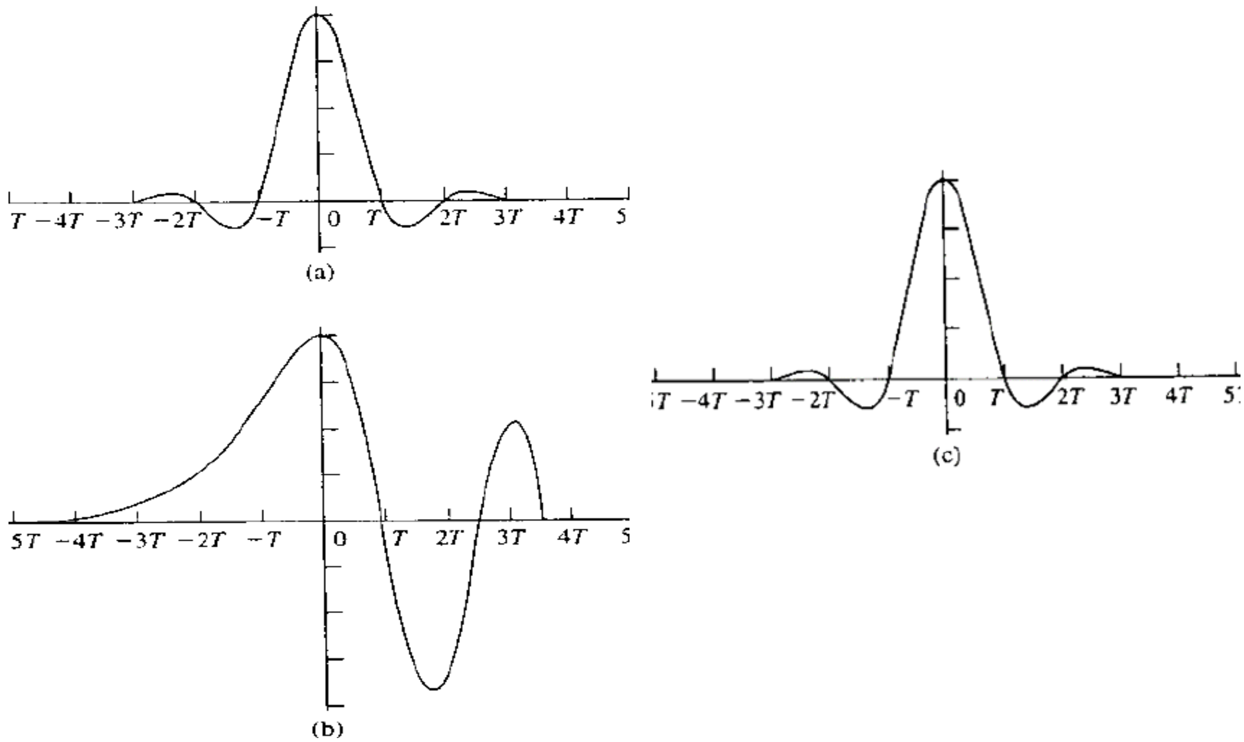


Figure 6.4: Effect of channel distortion: (a) channel input, (b) channel output, (c) equalizer output.

As an example, let us consider the intersymbol interference on a telephone channel. Figure 6.5 illustrates the measured average amplitude and delay as a function of frequency for a telephone channel of the switched telecommunications network. We observe that the usable band of the channel extends from about 300 Hz to about 3200 Hz. The corresponding impulse response of the average channel is shown in Figure 6.6. Its duration is about 10 ms. In comparison, the transmitted symbol rates on such a channel may be of the order of 2500 pulses or symbols per second. Hence, intersymbol interference might extend over 20 to 30 symbols.

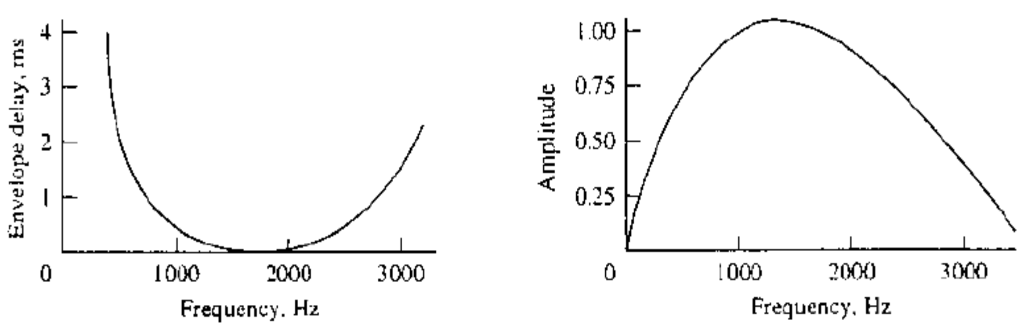


Figure 6.5: Average amplitude and delay characteristics of a medium-range telephone channel.

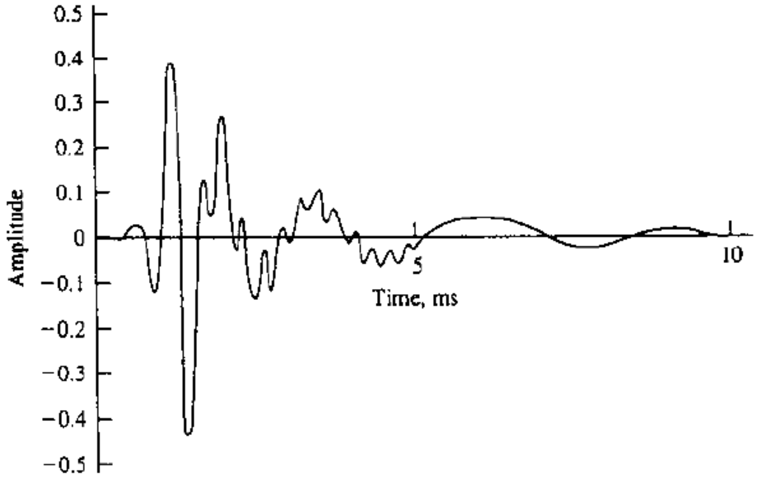


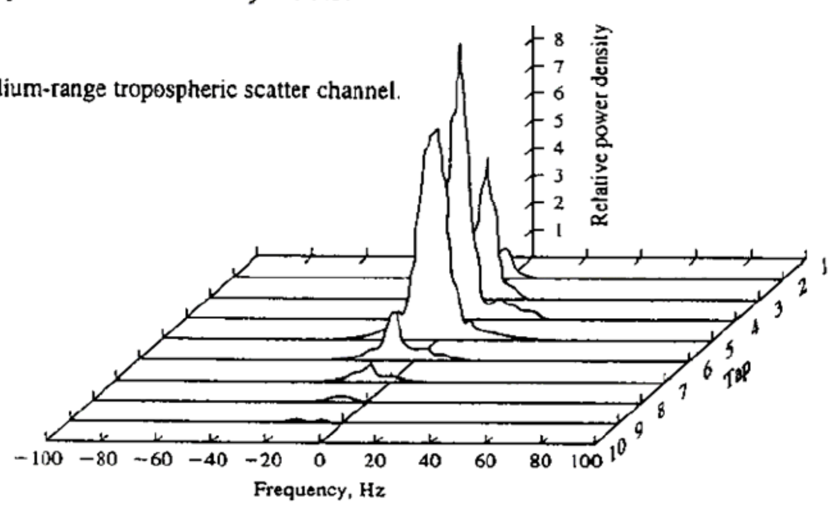
Figure 6.6: Impulse response of the average channel with amplitude and delay shown in Figure 6.5.

Besides telephone channels, there are other physical channels that exhibit some form of time dispersion and thus introduce intersymbol interference. Radio channels, such as shortwave ionospheric propagation (HF), tropospheric scatter, and mobile cellular radio are three examples of time-dispersive wireless channels. In these channels, time dispersion—and, hence, intersymbol interference—is the result of multiple propagation paths with different path delays. The number of paths and the relative time delays among the paths vary with time; for this reason, these radio channels are usually called *time-variant multipath channels*. The time-variant multipath conditions give rise to a wide variety of frequency

response characteristics. Consequently the frequency response characterization that is used for telephone channels is inappropriate for time-variant multipath channels. Instead, these radio channels are characterized statistically in terms of the scattering function, which, in brief, is a two-dimensional representation of the average received signal power as a function of relative time delay and Doppler frequency spread.

For illustrative purposes, a scattering function measured on a medium-range (150-mi) tropospheric scatter channel is shown in Figure 6.7. The total time duration (multipath spread) of the channel response is approximately $0.7\mu s$ on the average, and the spread between *half-power points* in Doppler frequency is a little less than 1 Hz on the strongest path and somewhat larger on the other paths. Typically, if transmission occurs at a rate of 10^7 symbols/second over such a channel, the multipath spread of $0.7\mu s$ will result in intersymbol interference that spans about seven symbols.

Figure 6.7: Scattering function of a medium-range tropospheric scatter channel.



6.4 Characterization of Intersymbol Interference

In a digital communication system, channel distortion causes intersymbol interference (ISI). In this section, we shall present a model that characterizes ISI. For simplicity, we assume

that the transmitted signal is a baseband PAM signal. However, this treatment is easily extended to carrier (linearly) modulated signals discussed in the next chapter.

The transmitted PAM signal is expressed as

$$s(t) = \sum_{n=0}^{\infty} a_n g(t - nT) \tag{6.4.1}$$

where $g(t)$ is the basic pulse shape that is selected to control the spectral characteristics of the transmitted signal, $\{a_n\}$ is the sequence of transmitted information symbols selected from a signal constellation consisting of M points, and T is the signal interval ($1/T$ is the symbol rate).

The signal $s(t)$ is transmitted over a baseband channel, which may be characterized by a frequency response $C(f)$. Consequently, the received signal can be represented as

$$r(t) = \sum_{n=0}^{\infty} a_n h(t - nT) + w(t) \tag{6.4.2}$$

where $h(t) = g(t) * c(t)$, $c(t)$ is the impulse response of the channel, $*$ denotes convolution, and $w(t)$ represents the additive noise in the channel. To characterize ISI, suppose that the received signal is passed through a receiving filter and then sampled at the rate $1/T$ samples/seconds. In general, the optimum filter at the receiver is matched to the received signal pulse $h(t)$. Hence, the frequency response of this filter is $H^*(f)$. We denote its output as

$$y(t) = \sum_{n=0}^{\infty} a_n x(t - nT) + v(t) \tag{6.4.3}$$

where $x(t)$ is the signal pulse response of the receiving filter, i.e., $X(f) = H(f)H^*(f) = |H(f)|^2$, and $v(t)$ is the response of the receiving filter to the noise $w(t)$. Now, if $y(t)$ is sampled at times $t = kT$, $k = 0, 1, 2, \ldots$, we have

$$y(kT) = \sum_{n=0}^{\infty} a_n x(kT - nT) + \nu(kT)$$

$$y_k = \sum_{n=0}^{\infty} a_n x_{k-n} + \nu_k, \qquad k = 0, 1, \dots \tag{6.4.4}$$

The sample values $\{y_k\}$ can be expressed as

$$y_k = x_0 \left(a_k + \frac{1}{x_0} \sum_{\substack{n=0 \\ n \neq k}}^{\infty} a_n x_{k-n} \right) + \nu_k, \qquad k = 0, 1, \dots \tag{6.4.5}$$

The term x_0 is an arbitrary scale factor, which we set equal to unity for convenience. Then

$$y_k = a_k + \sum_{\substack{n=0 \\ n \neq k}}^{\infty} a_n x_{k-n} + \nu_k \tag{6.4.6}$$

The term a_k represents the desired information symbol at the kth sampling instant, the term

$$\sum_{\substack{n=0 \\ n \neq k}}^{\infty} a_n x_{k-n} \tag{6.4.7}$$

represents the ISI, and ν_k is the additive noise at the kth sampling instant.

The amount of ISI and noise in a digital communications system can be viewed on an oscilloscope. For PAM signals, we can display the received signal $y(t)$ on the vertical input with the horizontal sweep rate set at $1/T$. The resulting oscilloscope display is called an *eye pattern* because of its resemblance to the human eye. For example, Figure 6.15 illustrates the eye patterns for binary and four-level PAM modulation. The effect of ISI is to cause the eye to close, thereby reducing the margin for additive noise to cause errors. Figure 6.16 graphically illustrates the effect of ISI in reducing the opening of a binary eye. Note that intersymbol interference distorts the position of the zero-crossings and causes a reduction in the eye opening. Thus, it causes the system to be more sensitive to a synchronization error.

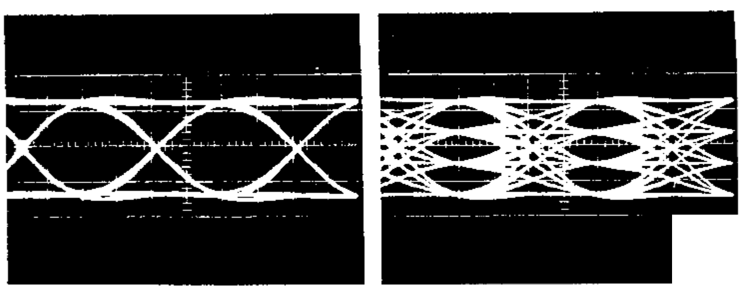
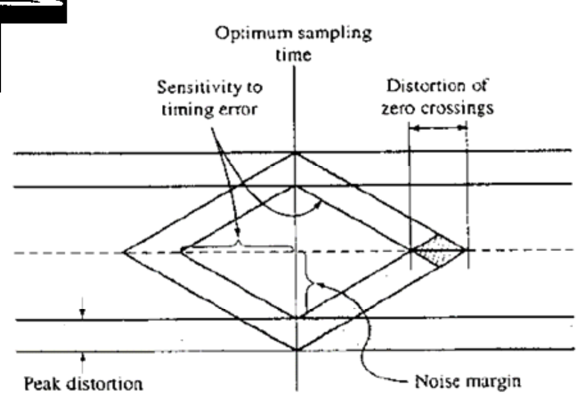


Figure 6.15: Examples of eye patterns for binary and quaternary amplitude shift keying (or PAM).

Figure 6.16: Effect of intersymbol interference on eye opening.



6.5 Communication System Design for Bandlimited Channels

In this section we consider the design of the transmitter and receiver filters that are suitable for a baseband bandlimited channel. Two cases are considered. In the first case, the design is based on transmitter and receiver filters that result in zero ISI. In the second case, the design is based on transmitter and receiver filters that have a specified (predetermined) amount of ISI. Thus, the second design approach leads to a controlled amount of ISI. The corresponding transmitted signals are called *partial response signals*. In both cases we assume that the channel is ideal; i.e., $A(f)$ and $\tau(f)$ are constant within the channel bandwidth W. For simplicity, we assume that $A(f) = 1$ and $\tau(f) = 0$.

6.5.1 Signal Design for Zero ISI

The design of bandlimited signals with zero ISI was a problem considered by Nyquist about 70 years ago. He demonstrated that a necessary and sufficient condition for a signal $x(t)$ to have zero ISI, i.e,

$$x(nT) = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases} \tag{6.5.1}$$

is that its Fourier transform $X(f)$ satisfy

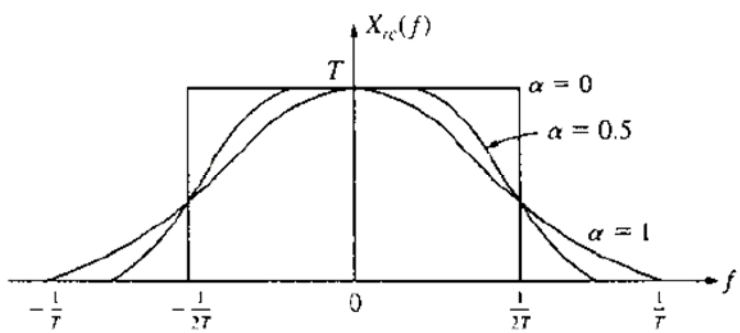
$$\sum_{m=-\infty}^{\infty} X\left(f + \frac{m}{T}\right) = T \tag{6.5.2}$$

where $1/T$ is the symbol rate.

In general, there are many signals that can be designed to have this property. One of the most commonly used signals in practice has a raised-cosine frequency response characteristic, which is defined as

$$X_{rc}(f) = \begin{cases} T, & 0 \le |f| \le \frac{(1-\alpha)}{2T} \\ \frac{T}{2}\left[1 + \cos\frac{\pi T}{\alpha}\left(|f| - \frac{1-\alpha}{2T}\right)\right], & \frac{1-\alpha}{2T} < |f| \le \frac{1+\alpha}{2T} \\ 0, & |f| > \frac{1+\alpha}{2T} \end{cases} \tag{6.5.3}$$

where α is called the *roll-off* factor, which takes values in the range $0 \le \alpha \le 1$, and $1/T$ is the symbol rate. The frequency response $X_{rc}(f)$ is illustrated in Figure 6.20(a) for $\alpha = 0$, $\alpha = \frac{1}{2}$, and $\alpha = 1$. Note that when $\alpha = 0$, $X_{rc}(f)$ reduces to an ideal "brick wall" physically nonrealizable frequency response with bandwidth occupancy $1/2T$. The

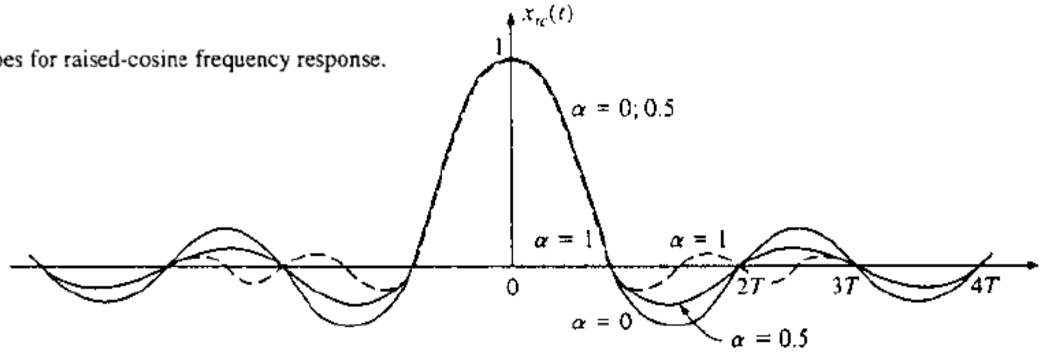


(a) Raised-cosine frequency response

Figure 6.20: Raised-cosine frequency response and corresponding pulse shape. (a) Raised-cosine frequency response. (b) Pulse shapes for raised-cosine frequency response.

Figure 6.20:

(b) Pulse shapes for raised-cosine frequency response.



(b) Pulse shapes for raised-cosine frequency response

frequency $1/2T$ is called the *Nyquist frequency*. For $\alpha > 0$, the bandwidth occupied by the desired signal $X_{rc}(f)$ beyond the Nyquist frequency $1/2T$ is called the *excess bandwidth*, usually expressed as a percentage of the Nyquist frequency. For example, when $\alpha = \frac{1}{2}$, the excess bandwidth is 50, and when $\alpha = 1$, the excess bandwidth is 100. The signal pulse $x_{rc}(t)$ having the raised-cosine spectrum is

$$x_{rc}(t) = \frac{\sin \pi t/T}{\pi t/T} \frac{\cos(\pi \alpha t/T)}{1 - 4\alpha^2 t^2/T^2} \tag{6.5.4}$$

Figure 6.20(b) illustrates $x_{rc}(t)$ for $\alpha = 0, \frac{1}{2}, 1$. Since $X_{rc}(f)$ satisfies (6.5.2), we note that $x_{rc}(t) = 1$ at $t = 0$ and $x_{rc}(t) = 0$ at $t = kT, k = \pm 1, \pm 2, \ldots$. Consequently, at the sampling instants $t = kT, k \neq 0$, there is no ISI from adjacent symbols when there

In an ideal channel, the transmitter and receiver filters are jointly designed for zero ISI at the desired sampling instants $t = nT$. Thus, if $G_T(f)$ is the frequency response of the transmitter filter and $G_R(f)$ is the frequency response of the receiver filter, then the product (cascade of the two filters) $G_T(f)G_R(f)$ is designed to yield zero ISI. For example, if the product $G_T(f)G_R(f)$ is selected as

$$G_T(f)G_R(f) = X_{rc}(f) \tag{6.5.5}$$

where $X_{rc}(f)$ is the raised-cosine frequency response characteristic, then the ISI at the sampling times $t = nT$ is zero.

6.5.2 Signal Design for Controlled ISI

As we have observed from our discussion of signal design for zero ISI, a transmit filter with excess bandwidth may be employed to realize practical transmitting and receiving filters for bandlimited channels. On the other hand, suppose we choose to relax the condition of zero ISI and thus achieve a symbol transmission in a bandwidth $W = 1/2T$ i.e., with no excess bandwidth. By allowing for a controlled amount of ISI, we can achieve the rate of $2W$ symbols/second.

We have already seen that the condition of zero ISI is $x(nT) = 0$ for $n \neq 0$. However, suppose that we design the bandlimited signal to have controlled ISI at one time instant. This means that we allow one additional nonzero value in the samples $\{x(nT)\}$. The ISI that we introduce is deterministic, or "controlled"; hence, it can be taken into account at the receiver, as discussed below.

In general, a signal $x(t)$ that is bandlimited to W hertz, i.e.,

$$X(f) = 0, \qquad |f| > W \tag{6.5.10}$$

can be represented as

$$x(t) = \sum_{n=-\infty}^{\infty} x\left(\frac{n}{2W}\right) \frac{\sin 2\pi W(t - n/2W)}{2\pi W(t - n/2W)} \tag{6.5.11}$$

This representation follows from the sampling theorem for bandlimited signals. The spectrum of the bandlimited signal is

$$\begin{aligned} X(f) &= \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t}\, dt \\ &= \begin{cases} \dfrac{1}{2W} \displaystyle\sum_{n=-\infty}^{\infty} x\left(\frac{n}{2W}\right) e^{-jn\pi f/W}, & |f| \le W \\ 0, & |f| > W \end{cases} \end{aligned} \tag{6.5.12}$$

One special case that leads to physically realizable transmitting and receiving filters is specified by the samples

$$x\left(\frac{n}{2W}\right) \equiv x(nT) = \begin{cases} 1, & n = 0, 1 \\ 0, & \text{otherwise} \end{cases} \tag{6.5.13}$$

The corresponding signal spectrum is

$$\begin{aligned} X(f) &= \begin{cases} \dfrac{1}{2W}\left[1 + e^{-j\pi f/W}\right], & |f| < W \\ 0, & \text{otherwise} \end{cases} \\ &= \begin{cases} \dfrac{1}{W} e^{-j2\pi f/W} \cos\left(\dfrac{\pi f}{2W}\right), & |f| < W \\ 0, & \text{otherwise} \end{cases} \end{aligned} \tag{6.5.14}$$

Therefore, $x(t)$ is given by

$$x(t) = \operatorname{sinc}(2Wt) + \operatorname{sinc}(2Wt - 1) \tag{6.5.15}$$

where $\operatorname{sinc}(t) = \sin \pi t / \pi t$. This pulse is called a *duobinary signal pulse*. It is illustrated along with its magnitude spectrum in Figure 6.23. We note that the spectrum decays to zero smoothly, which means that physically realizable filters can be designed that approximate this spectrum very closely. Thus, a symbol rate of $2W$ is achieved.

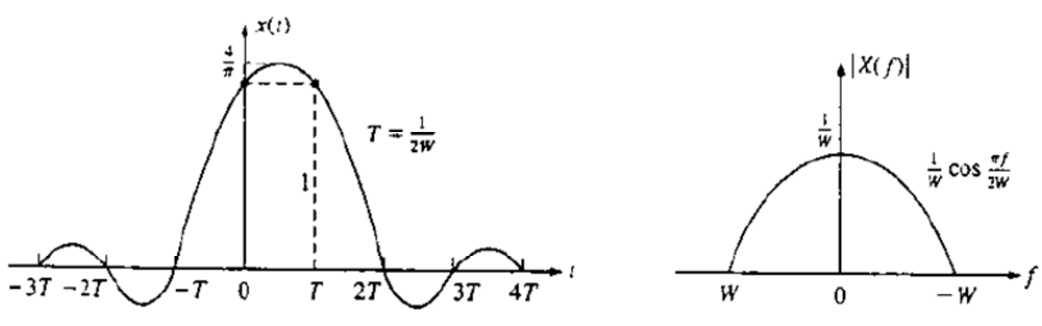


Figure 6.23: Duobinary signal pulse and its spectrum.

Another special case that leads to physically realizable transmitting and receiving filters is specified by the samples

$$x\left(\frac{n}{2W}\right) = x(nT) = \begin{cases} 1, & n = 1 \\ -1, & n = -1 \\ 0, & \text{otherwise} \end{cases} \tag{6.5.16}$$

The corresponding pulse $x(t)$ is given as

$$x(t) = \text{sinc}(2Wt + 1) - \text{sinc}\,(2Wt - 1) \tag{6.5.17}$$

and its spectrum is

$$X(f) = \begin{cases} \dfrac{1}{2W}\left(e^{j\pi f/W} - e^{-j\pi f/W}\right) = \dfrac{j}{W}\sin\dfrac{\pi f}{W}, & |f| \le W \\ 0, & |f| > W \end{cases} \tag{6.5.18}$$

This pulse and its magnitude spectrum are illustrated in Figure 6.24. It is called a *modified duobinary signal pulse.* It is interesting to note that the spectrum of this signal has a zero at $f = 0$, making it suitable for transmission over a channel that does not pass dc.

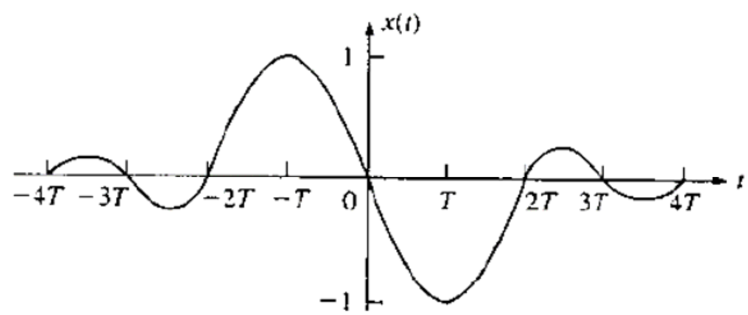
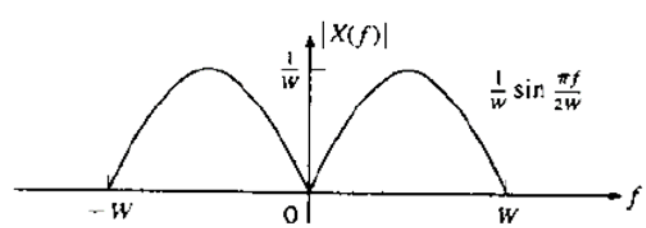


Figure 6.24: Modified duobinary signal pulse and its spectrum.



One can obtain other interesting and physically realizable filter characteristics by selecting different values for the samples $\{x(n/2W)\}$ and more than two nonzero samples. However, as we select more nonzero samples, the problem of unraveling the controlled ISI becomes more cumbersome and impractical.

The signals obtained when controlled ISI is purposely introduced by selecting two or more nonzero samples from the set $\{x(n/2W)\}$ are called *partial response signals.* The resulting signal pulses allow us to transmit information symbols at the Nyquist rate of $2W$ symbols per second. Thus greater bandwidth efficiency is obtained compared to raised cosine signal pulses.

6.5.3 Precoding for Detection of Partial Response Signals

For the duobinary signal pulse, $x(nT) = 1$ for $n = 0, 1$ and 0 otherwise. Hence, the samples of the output of the receiver filter $G_R(f)$ are expressed as

$$\begin{aligned} y_k &= a_k + a_{k-1} + \nu_k \\ &= b_k + \nu_k \end{aligned} \tag{6.5.22}$$

where $\{a_k\}$ is the transmitted sequence of amplitudes, $\{\nu_k\}$ is a sequence of additive Gaussian noise samples, and $b_k = a_k + a_{k-1}$. Let us ignore the noise for the moment and consider the binary case where $a_k = \pm 1$ with equal probability. Then, b_k takes one of three possible values, namely, $b_k = -2, 0, 2$ with corresponding probabilities $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{1}{4}$. If a_{k-1} is the detected signal from the $(k-1)$st signaling interval, its effect on b_k, the received signal in the kth signaling interval, can be eliminated by subtraction, thus allowing a_k to be detected. The process can be repeated sequentially for every received symbol.

The major problem with this procedure is that errors arising from the additive noise tend to propagate. For example, if a_{k-1} is detected in error, its effect on a_k is not eliminated; in effect, it is reinforced by the incorrect subtraction. Consequently, the detection of a_k is also likely to be in error.

Error propagation can be prevented by *precoding* the data at the transmitter instead of eliminating the controlled ISI by subtraction at the receiver. The precoding is performed on the binary data sequence prior to modulation. From the data sequence $\{D_k\}$ of 1's and 0's

that is to be transmitted, a new sequence $\{p_k\}$, called the *precoded sequence*, is generated. For the duobinary signal, the precoded sequence is defined as

$$p_k = D_k \ominus p_{k-1}, \qquad k = 1, 2, \ldots \tag{6.5.23}$$

where $\ominus$ denotes modulo-2 subtraction.[1] Then the transmitted signal amplitude is $a_k = -1$ if $p_k = 0$ and $a_k = 1$ if $p_k = 1$. That is,

$$a_k = 2p_k - 1 \tag{6.5.24}$$

The noise-free samples at the output of the receiving filter are given by

$$\begin{aligned} b_k &= a_k + a_{k-1} \\ &= (2p_k - 1) + (2p_{k-1} - 1) \\ &= 2(p_k + p_{k-1} - 1) \end{aligned} \tag{6.5.25}$$

Consequently,

$$p_k + p_{k-1} = \frac{1}{2}b_k + 1 \tag{6.5.26}$$

Since $D_k = p_k \oplus p_{k-1}$, it follows that the data sequence $\{D_k\}$ is obtained from $\{b_k\}$ using the relation

[1] Although this operation is identical to modulo-2 addition, it is convenient to view the precoding operation for duobinary in terms of modulo-2 subtraction.

$$D_k = \frac{1}{2}b_k + 1 \pmod 2 \tag{6.5.27}$$

Therefore, if $b_k = \pm 2$, then $D_k = 0$, and if $b_k = 0$, then $D_k = 1$. An example that illustrates the precoding and decoding operations is given in Table 6.1.

In the presence of additive noise, the sampled outputs from the receiving filter are given by (6.5.22). In this case $y_k = b_k + \nu_k$ is compared with the two thresholds set at $+1$ and -1. The data sequence $\{D_k\}$ is obtained according to the detection rule

$$D_k = \begin{cases} 1, & |y_k| < 1 \\ 0, & |y_k| \geq 1 \end{cases} \tag{6.5.28}$$

Thus precoding the data allows us to perform symbol-by-symbol detection at the receiver without the need for subtraction of previously detected symbols.

The extension from binary PAM to multilevel PAM using duobinary pulses is straightforward. The M-level transmitted sequence $\{a_k\}$ results in a (noise-free) received sequence

$$b_k = a_k + a_{k-1}, \qquad k = 1, 2, 3, \ldots \tag{6.5.29}$$

which has $2M - 1$ possible equally spaced amplitude levels. The amplitude levels for the sequence $\{a_k\}$ are determined from the relation

$$a_k = 2p_k - (M - 1) \tag{6.5.30}$$

where $\{p_k\}$ is the precoded sequence that is obtained from an M-level data sequence $\{D_k\}$ according to the relation

$$p_k = D_k \ominus p_{k-1} \pmod M \tag{6.5.31}$$

where the possible values of the data sequence $\{D_k\}$ are $0, 1, 2, \ldots, M$.

In the absence of noise, the samples at the output of the receiving filter may be expressed as

$$\begin{aligned} b_k &= a_k + a_{k-1} \\ &= [2p_k - (M-1)] + \left[2p_{k-1} - (M-1)\right] \\ &= 2\left[p_k + p_{k-1} - (M-1)\right] \end{aligned} \tag{6.5.32}$$

Hence,

$$p_k + p_{k-1} = \frac{1}{2}b_k + (M - 1) \tag{6.5.33}$$

Since $D_k = p_k + p_{k-1} \pmod M$, it follows that the transmitted data $\{D_k\}$ are recovered from the received sequence $\{b_k\}$ by means of the relation

$$D_k = \frac{1}{2}b_k + (M - 1) \pmod M \tag{6.5.34}$$

In the case of modified duobinary pulse, the received signal samples at the output of the receiving filter $G_R(f)$ are expressed as

$$\begin{aligned} y_k &= a_k - a_{k-2} + \nu_k \\ &= b_k + \nu_k \end{aligned} \tag{6.5.35}$$

The precoder for the modified duobinary pulse produces the sequence $\{p_k\}$ from the data sequence $\{D_k\}$ according to the relation

$$p_k = D_k \oplus p_{k-2} \pmod M \tag{6.5.36}$$

From these relations, it is easy to show that the detection rule for the recovering of the data sequence $\{D_k\}$ from $\{b_k\}$ in the absence of noise is

$$D_k = \frac{1}{2} b_k \pmod M \tag{6.5.37}$$

6.6 Linear Equalizers

The most common type of channel equalizer used in practice to reduce ISI is a linear FIR filter with adjustable coefficients $\{c_i\}$, as shown in Figure 6.27.

On channels whose frequency response characteristics are unknown but time-invariant, we may measure the channel characteristics and adjust the parameters of the equalizer; once adjusted, the parameters remain fixed during the transmission of data. Such equalizers are called *preset equalizers*. On the other hand, *adaptive equalizers* update their parameters on a periodic basis during the transmission of data, so they are capable of tracking a slowly time-varying channel response.

First, let us consider the design characteristics for a linear equalizer from a frequency domain viewpoint. Figure 6.28 shows a block diagram of a system that employs a linear filter as a channel equalizer.

The demodulator consists of a receiver filter with frequency response $G_R(f)$ in cascade with a channel equalizing filter that has a frequency response $G_E(f)$. As indicated in the previous section, the receiver filter response $G_R(f)$ is matched to the transmitter response, i.e., $G_R(f) = G_T^*(f)$, and the product $G_R(f)G_T(f)$ is usually designed so that either there is zero ISI at the sampling instants as, for example, when $G_R(t)G_T(f) = X_{rc}(f)$, or controlled ISI for partial response signals.

For the system shown in Figure 6.28, in which the channel frequency response is not ideal, the desired condition for zero ISI is

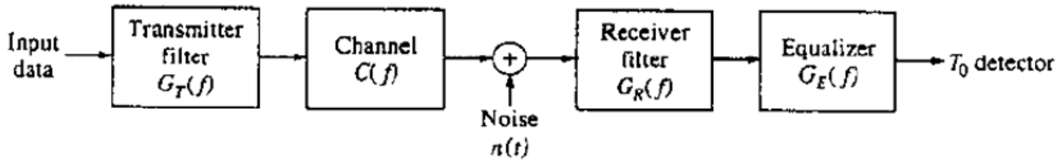


Figure 6.28: Block diagram of a system with an equalizer.

$$G_T(f)C(f)G_R(f)G_E(f) = X_{rc}(f) \tag{6.6.1}$$

where $X_{rc}(f)$ is the desired raised-cosine spectral characteristic. Since $G_T(f)\,G_R(f) = X_{rc}(f)$ by design, the frequency response of the equalizer that compensates for the channel distortion is

$$G_E(f) = \frac{1}{C(f)} = \frac{1}{|C(f)|}e^{-j\theta_c(f)} \tag{6.6.2}$$

Thus, the amplitude response of the equalizer is $|G_E(f)| = 1/|C(f)|$, and its phase response is $\theta_E(f) = -\theta_C(f)$. In this case, the equalizer is said to be the *inverse channel filter* to the channel response.

We note that the inverse channel filter completely eliminates ISI caused by the channel. Since it forces the ISI to be zero at the sampling instants $t = kT$ for $k = 0, 1, \ldots,$ the equalizer is called a *zero-forcing equalizer*. Hence, the input to the detector is simply

$$z_k = a_k + \eta_k, \qquad k = 0, 1, \ldots \tag{6.6.3}$$

where η_k represents the additive noise and a_k is the desired symbol.

In practice, the ISI caused by channel distortion is usually limited to a finite number of symbols on either side of the desired symbol. Hence, the number of terms that constitute the ISI in the summation given by (6.4.7) is finite. As a consequence, in practice the channel equalizer is implemented as a finite-duration impulse response (FIR) filter, or transversal filter, with adjustable tap coefficients $\{c_n\}$, as illustrated in Figure 6.27. The time delay τ between adjacent taps may be selected as large as T, the symbol interval, in which case the FIR equalizer is called a *symbol-spaced equalizer*. In this case the input to the equalizer is the sampled sequence given by (6.4.6). However, we note that when the symbol rate $1/T < 2W$, frequencies in the received signal above the folding frequency $1/T$ are aliased into frequencies below $1/T$. In this case, the equalizer compensates for the aliased channel-distorted signal.

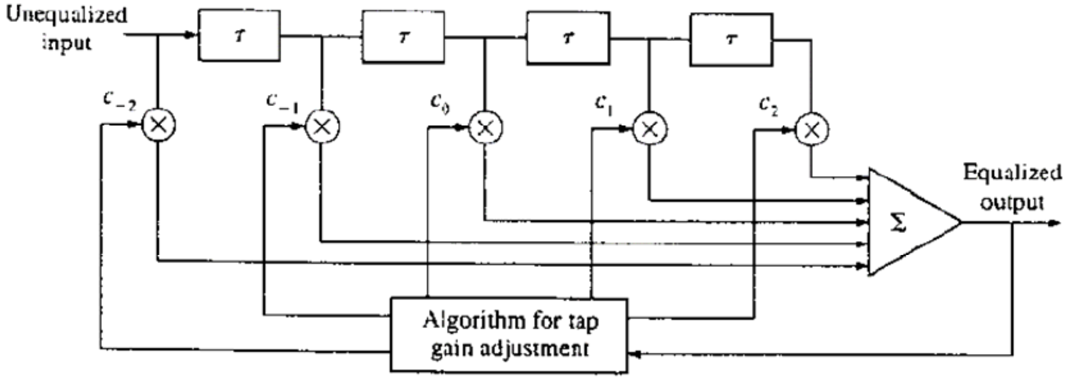


Figure 6.27: Linear transversal filter.

On the other hand, when the time delay τ between adjacent taps is selected such that $1/\tau \geq 2W > 1/T$, no aliasing occurs; hence the inverse channel equalizer compensates for the true channel distortion. Since $\tau < T$, the channel equalizer is said to have *fractionally spaced taps*, and it is called a *fractionally spaced equalizer*. In practice, τ is often selected at $\tau = T/2$. Notice that, in this case, the sampling rate at the input to the filter $G(f)$ is $\frac{2}{T}$.

The impulse response of the FIR equalizer is

$$g_E(t) = \sum_{n=-K}^{K} c_n \delta(t - n\tau) \tag{6.6.4}$$

and the corresponding frequency response is

$$G_E(f) = \sum_{n=-K}^{K} c_n e^{-j2\pi f n\tau} \tag{6.6.5}$$

where $\{c_n\}$ are the $2K + 1$ equalizer coefficients and K is chosen sufficiently large so that the equalizer spans the length of the ISI, i.e., $2K + 1 \geq L$, where L is the number of signal samples spanned by the ISI. Since $X(f) = G_T(f)C(f)G_R(f)$ and $x(t)$ is the signal pulse corresponding to $X(f)$, the equalized output signal pulse is

$$q(t) = \sum_{n=-K}^{K} c_n x(t - n\tau) \tag{6.6.6}$$

The zero-forcing condition can now be applied to the samples of $q(t)$ taken at times $t = mT$. These samples are

$$q(mT) = \sum_{n=-K}^{K} c_n x(mT - n\tau), \qquad m = 0, \pm 1, \ldots, \pm K \tag{6.6.7}$$

Since there are $2K + 1$ equalizer coefficients, we can control only $2K + 1$ sampled values of $q(t)$. Specifically, we may force the conditions

$$\begin{aligned} q(mT) &= \sum_{n=-K}^{K} c_n x(mT - n\tau) \\ &= \begin{cases} 1, & m = 0 \\ 0, & m = \pm 1, \pm 2, \ldots, \pm K \end{cases} \end{aligned} \tag{6.6.8}$$

which may be expressed in matrix form as $\boldsymbol{Xc} = \boldsymbol{q}$, where $\boldsymbol{X}$ is a $(2K + 1) \times (2K + 1)$ matrix with elements $x(mT - n\tau)$, $\boldsymbol{c}$ is the $(2K + 1)$ coefficient vector, and $\boldsymbol{q}$ is the $(2K + 1)$ column vector with one nonzero element. Thus, we obtain a set of $2K + 1$ linear equations for the coefficients of the zero-forcing equalizer.

We should emphasize that the FIR zero-forcing equalizer does not completely eliminate ISI because it has a finite length. However, as K is increased, the residual ISI can be reduced and in the limit of $K \rightarrow \infty$, the ISI is completely eliminated.

One drawback to the zero-forcing equalizer is that it ignores the presence of additive noise. As a consequence, its use may result in significant noise enhancement. This is easily seen by noting that in a frequency range where $C(f)$ is small, the channel equalizer $G_E(f) = 1/C(f)$ compensates by placing a large gain in that frequency range. Consequently, the noise in that frequency range is greatly enhanced. An alternative is to relax the zero ISI condition and select the channel equalizer characteristic such that the combined power in the residual ISI and the additive noise at the output of the equalizer is minimized. A channel equalizer that is optimized based on the minimum mean-square error (MMSE) criterion accomplishes the desired goal.

To elaborate, let us consider the noise corrupted output of the FIR equalizer, which is

$$z(t) = \sum_{n=-K}^{K} c_n y(t - n\tau) \tag{6.6.12}$$

where $y(t)$ is the input to the equalizer, given by (6.4.3). The equalizer output is sampled at times $t = mT$. Thus, we obtain

$$z(mT) = \sum_{n=-K}^{K} c_n y(mT - n\tau) \tag{6.6.13}$$

The desired response at the output of the equalizer at $t = mT$ is the transmitted symbol a_m. The error is defined as the difference between a_m and $z(mT)$. Then, the mean-square error (MSE) between the actual output sample $z(mT)$ and the desired values a_m is[2]

$$\begin{aligned}
\text{MSE} &= E\,|z(mT) - a_m|^2 \\
&= E\left[\left|\sum_{n=-K}^{K} c_n y(mT - n\tau) - a_m\right|^2\right] \\
&= \sum_{n=-K}^{K}\sum_{k=-K}^{K} c_n c_k R_y(n-k) - 2\sum_{k=-K}^{K} c_k R_{ay}(k) + E(|a_m|^2)
\end{aligned} \tag{6.6.14}$$

where the correlations are defined as

$$\begin{aligned}
R_y(n-k) &= E[y^*(mT - n\tau)y(mT - k\tau)] \\
R_{ay}(k) &= E[y(mT - k\tau)a_m^*]
\end{aligned} \tag{6.6.15}$$

[2] In this development we allow the signals $z(t)$ and $y(t)$ to be complex-valued and the data sequence also to be complex-valued.

and the expectation is taken with respect to the random information sequence $\{a_m\}$ and the additive noise.

The minimum MSE solution is obtained by differentiating (6.6.14) with respect to the equalizer coefficients $\{c_n\}$. Thus we obtain the necessary conditions for the minimum MSE as

$$\sum_{n=-K}^{K} c_n R_y(n-k) = R_{ay}(k), \qquad k = 0, \pm 1, \pm 2, \ldots, \pm K \tag{6.6.16}$$

These are the $2K+1$ linear equations for the equalizer coefficients. In contrast to the zero-forcing solution described previously, these equations depend on the statistical properties (the autocorrelation) of the noise as well as the ISI through the autocorrelation $R_y(n)$.

In practice, the autocorrelation matrix $R_y(n)$ and the cross-correlation vector $R_{ay}(n)$ are unknown a priori. However, these correlation sequences can be estimated by transmitting a test signal over the channel and using the time-average estimates

$$\hat{R}_y(n) = \frac{1}{K}\sum_{k=1}^{K} y^*(kT - n\tau)y(kT)$$
$$\hat{R}_{ay}(n) = \frac{1}{K}\sum_{k=1}^{K} y(kT - n\tau)a_k^* \tag{6.6.17}$$

in place of the ensemble averages to solve for the equalizer coefficients given by (6.6.16).

6.6.1 Adaptive Linear Equalizers

We have shown that the tap coefficients of a linear equalizer can be determined by solving a set of linear equations. In the zero-forcing optimization criterion, the linear equations are given by (6.6.8). On the other hand, if the optimization criterion is based on minimizing the MSE, the optimum equalizer coefficients are determined by solving the set of linear equations given by (6.6.16).

In both cases, we may express the set of linear equations in the general matrix form

$$\boldsymbol{B}\boldsymbol{c} = \boldsymbol{d} \tag{6.6.18}$$

where $\boldsymbol{B}$ is a $(2K+1) \times (2K+1)$ matrix, $\boldsymbol{c}$ is a column vector representing the $2K+1$ equalizer coefficients, and $\boldsymbol{d}$ is a $(2K+1)$-dimensional column vector. The solution of (6.6.18) yields

$$\boldsymbol{c}_{\text{opt}} = \boldsymbol{B}^{-1}\boldsymbol{d} \tag{6.6.19}$$

In practical implementations of equalizers, the solution of (6.6.18) for the optimum coefficient vector is usually obtained by an iterative procedure that avoids the explicit computation of the inverse of the matrix $\boldsymbol{B}$. The simplest iterative procedure is the method of steepest descent, in which one begins by choosing arbitrarily the coefficient vector $\mathbf{c}$, say, $\boldsymbol{c}_0$. This initial choice of coefficient vector $\boldsymbol{c}_0$ corresponds to a point on the criterion function that is being optimized. For example, in the case of the MSE criterion, the initial guess c_0 corresponds to a point on the quadratic MSE surface in the $(2K+1)$-dimensional space of coefficients. The gradient vector, defined as g_0, which is the derivative of the MSE with respect to the $2K+1$ filter coefficients, is then computed at this point on the criterion surface, and each tap coefficient is changed in the direction opposite to its corresponding gradient component. The change in the jth tap coefficient is proportional to the size of the jth gradient component.

For example, the gradient vector, denoted as g_k, for the MSE criterion, found by taking the derivatives of the MSE with respect to each of the $2K+1$ coefficients, is

$$g_k = Bc_k - d, \qquad k = 0, 1, 2, \ldots \tag{6.6.20}$$

Then the coefficient vector c_k is updated according to the relation

$$c_{k+1} = c_k - \Delta g_k \tag{6.6.21}$$

Adaptive channel equalization is required for channels whose characteristics change with time. In such a case, the ISI varies with time. The channel equalizer must track such time variations in the channel response and adapt its coefficients to reduce the ISI. In the context of the above discussion, the optimum coefficient vector c_{opt} varies with time due to time variations in the matrix B and, for the case of the MSE criterion, time variations in the vector d. Under these conditions, the iterative method described above can be modified to use estimates of the gradient components. Thus, the algorithm for adjusting the equalizer tap coefficients may be expressed as

$$\hat{c}_{k+1} = \hat{c}_k - \Delta \hat{g}_k \tag{6.6.22}$$

where $\hat{g}_k$ denotes an estimate of the gradient vector g_k and $\hat{c}_k$ denotes the estimate of the tap coefficient vector.

In the case of the MSE criterion, the gradient vector g_k given by (6.6.20) may also be expressed as

$$g_k = -E(e_k y_k^*)$$

An estimate $\hat{g}_k$ of the gradient vector at the kth iteration is computed as

$$\hat{g}_k = -e_k y_k^* \tag{6.6.23}$$

where e_k denotes the difference between the desired output from the equalizer at the kth time instant and the actual output $z(kT)$ and y_k denotes the column vector of $2K+1$ received signal values contained in the equalizer at time instant k. The *error signal* e_k is expressed as

$$e_k = a_k - z_k \tag{6.6.24}$$

where $z_k = z(kT)$ is the equalizer output given by (6.6.13) and a_k is the desired symbol. Hence, by substituting (6.6.23) into (6.6.22), we obtain the adaptive algorithm for optimizing the taps coefficients (based on the MSE criterion) as

$$\hat{c}_{k+1} = \hat{c}_k + \Delta e_k y_k^* \tag{6.6.25}$$

Since an estimate of the gradient vector is used in (6.6.25), the algorithm is called a *stochastic gradient algorithm*. It is also known as the *LMS algorithm*.

A block diagram of an adaptive equalizer that adapts its tap coefficients according to (6.6.25) is illustrated in Figure 6.32. Note that the difference between the desired output a_k and the actual output z_k from the equalizer is used to form the error signal e_k. This error is scaled by the step-size parameter Δ, and the scaled error signal Δe_k multiples the received signal values $\{y(kT - n\tau)\}$ at the $2K + 1$ taps. The products $\Delta e_k y^*(kT - n\tau)$ at the $2K + 1$ taps are then added to the previous values of the tap coefficients to obtain the updated tap coefficients, according to (6.6.25). This computation is repeated as each new signal sample is received. Thus, the equalizer coefficients are updated at the symbol rate.

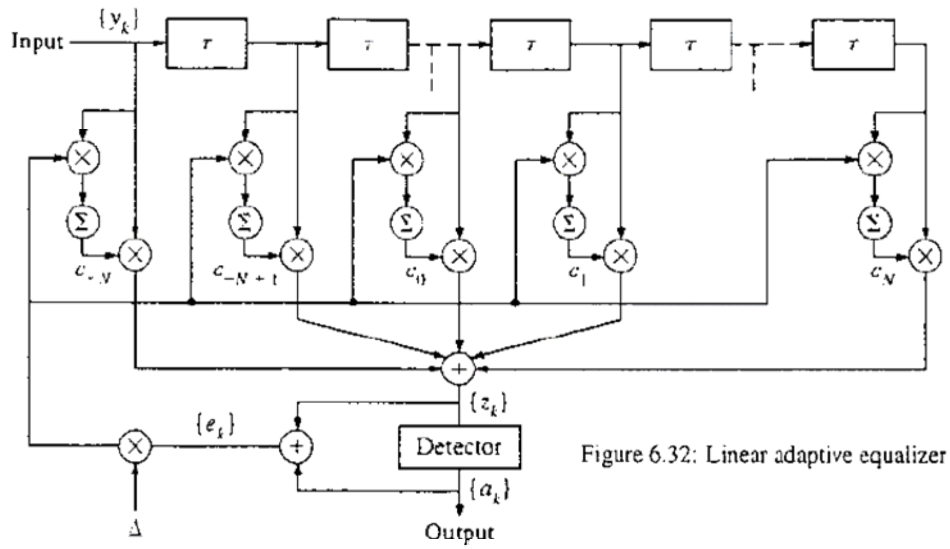


Figure 6.32: Linear adaptive equalizer based on the MSE criterion.

Initially, the adaptive equalizer is trained by the transmission of a known pseudorandom sequence $\{a_m\}$ over the channel. At the demodulator, the equalizer employs the known sequence to adjust its coefficients. Upon initial adjustment, the adaptive equalizer switches from a *training mode* to a *decision-directed mode*, in which case the decisions at the output of the detector are sufficiently reliable so that the error signal is formed by computing the difference between the detector output and the equalizer output, i.e.,

$$e_k = \hat{a}_k - z_k \tag{6.6.26}$$

where $\hat{a}_k$ is the output of the detector. In general, decision errors at the output of the detector occur infrequently; consequently, such errors have little effect on the performance of the tracking algorithm given by (6.6.25).

A rule of thumb for selecting the step-size parameter in order to ensure convergence and good tracking capabilities in slowly varying channels is

$$\Delta = \frac{1}{5(2K+1)P_R} \tag{6.6.27}$$

where P_R denotes the received signal-plus-noise power, which can be estimated from the received signal.

Although we have described in some detail the operation of an adaptive equalizer that is optimized on the basis of the MSE criterion, the operation of an adaptive equalizer based on the zero-forcing method is very similar. The major difference lies in the method for estimating the gradient vectors g_k at each iteration. A block diagram of an adaptive zero-forcing equalizer is shown in Figure 6.34.

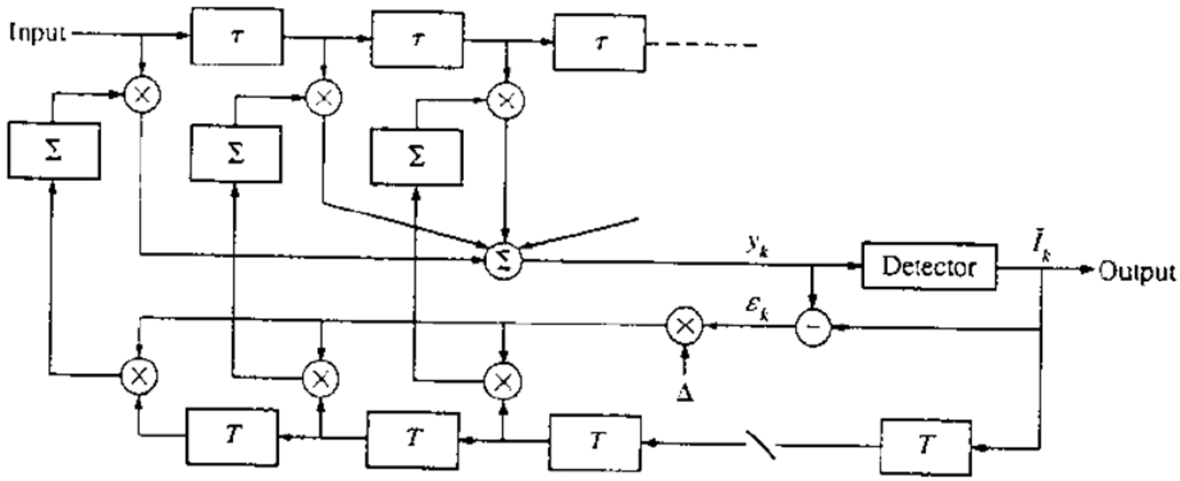


Figure 6.34: An adaptive zero-forcing equalizer.

6.7 Nonlinear Equalizers

The linear filter equalizers described above are very effective on channels, such as wire line telephone channels, where the ISI is not severe. The severity of the ISI is directly related to the spectral characteristics of the channel and not necessarily to the time span of the ISI. For example, consider the ISI resulting from two channels, illustrated in Figure 6.35. The time span for the ISI in Channel A is five symbol intervals on each side of the desired signal

component, which has a value of 0.72. On the other hand, the time span for the ISI in Channel B is one symbol interval on each side of the desired signal component, which has a value of 0.815. The energy of the total response is normalized to unity for both channels.

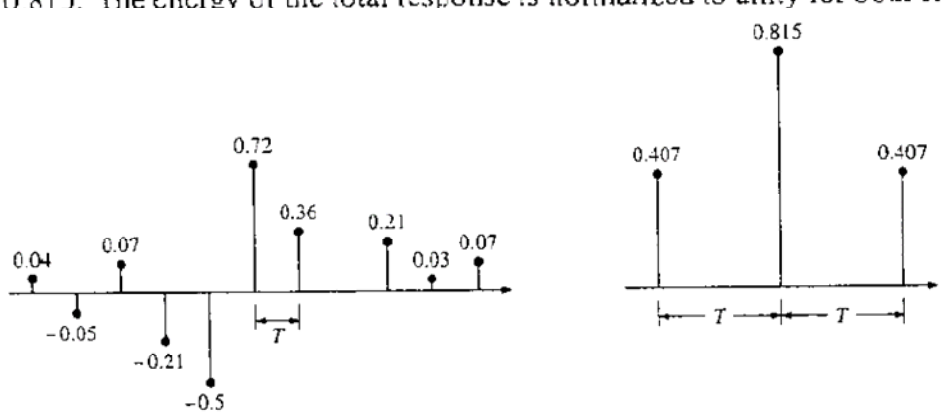


Figure 6.35: Two channels with ISI.

In spite of the shorter ISI span, channel B results in more severe ISI. This is evidenced in the frequency response characteristics of these channels, which are shown in Figure 6.36. We observe that channel B has a spectral null (the frequency response $C(f) = 0$ for some frequencies in the band $|f| \leq W$) at $f = 1/2T$, whereas this does not occur in the case of channel A. Consequently, a linear equalizer will introduce a large gain in its frequency response to compensate for the channel null. Thus, the noise in channel B will be enhanced much more than in channel A. This implies that the performance of the linear equalizer for channel B will be sufficiently poorer than that for channel A. In general, the basic limitation of a linear equalizer is that it performs poorly on channels having spectral nulls. Such channels are often encountered in radio communications, such as ionospheric transmission at frequencies below 30 MHz and mobile radio channels, such as those used for cellular radio communications.

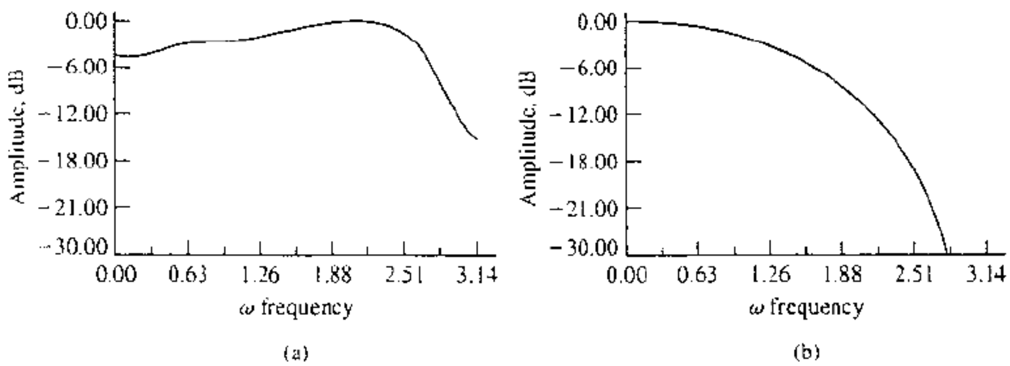


Figure 6.36: Amplitude spectra for (a) channel A shown in Figure 6.35(a) and (b) channel B shown in Figure 6.35(b).

A *decision-feedback equalizer* (DFE) is a nonlinear equalizer that employs previous decisions to eliminate the ISI caused by previously detected symbols on the current symbol to be detected. A simple block diagram for a DFE is shown in Figure 6.37. The DFE consists of two filters. The first filter is called a *feedforward filter*; it is generally a fractionally spaced FIR filter with adjustable tap coefficients. This filter is identical in form to the linear equalizer described above. Its input is the received filtered signal $y(t)$ sampled at some rate that is a multiple of the symbol rate, e.g., at rate $2/T$. The second filter is a *feedback filter*. It is implemented as an FIR filter with symbol-spaced taps having adjustable coefficients. Its input is the set of previously detected symbols. The output of the feedback filter is

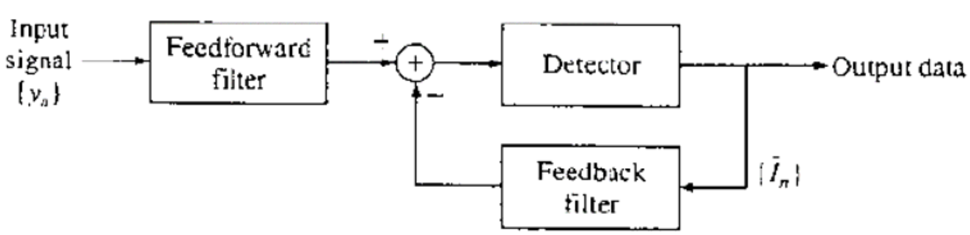


Figure 6.37: Block diagram of a DFE.

subtracted from the output of the feedforward filter to form the input to the detector. Thus, we have

$$z_m = \sum_{n=1}^{N_1} c_n y(mT - n\tau) - \sum_{n=1}^{N_2} b_n \tilde{a}_{m-n}$$

where $\{c_n\}$ and $\{b_n\}$ are the adjustable coefficients of the feedforward and feedback filters, respectively, $\tilde{a}_{m-n}$, $n = 1, 2, \ldots, N_2$, are the previously detected symbols, N_1 is the length of the feedforward filter, and N_2 is the length of the feedback filter. Based on the input z_m, the detector determines which of the possible transmitted symbols is closest in distance to the input signal a_m. Thus it makes its decision and outputs $\tilde{a}_m$. What makes the DFE nonlinear is the nonlinear characteristic of the detector that provides the input to the feedback filter.

The tap coefficients of the feedforward and feedback filters are selected to optimize some desired performance measure. For mathematical simplicity, the MSE criterion is usually applied, and a stochastic gradient algorithm is commonly used to implement an adaptive DFE. Figure 6.38 illustrates the block diagram of an adaptive DFE whose tap coefficients are adjusted by means of the LMS stochastic gradient algorithm.

We should mention that decision errors from the detector that are fed to the feedback filter have a small effect on the performance of the DFE. In general, a small loss in performance of 1 to 2 dB is possible at error rates below 10^{-2}, but the decision errors in the feedback filters are not catastrophic.

Although the DFE outperforms a linear equalizer, it is not the optimum equalizer from the viewpoint of minimizing the probability of error in the detection of the information

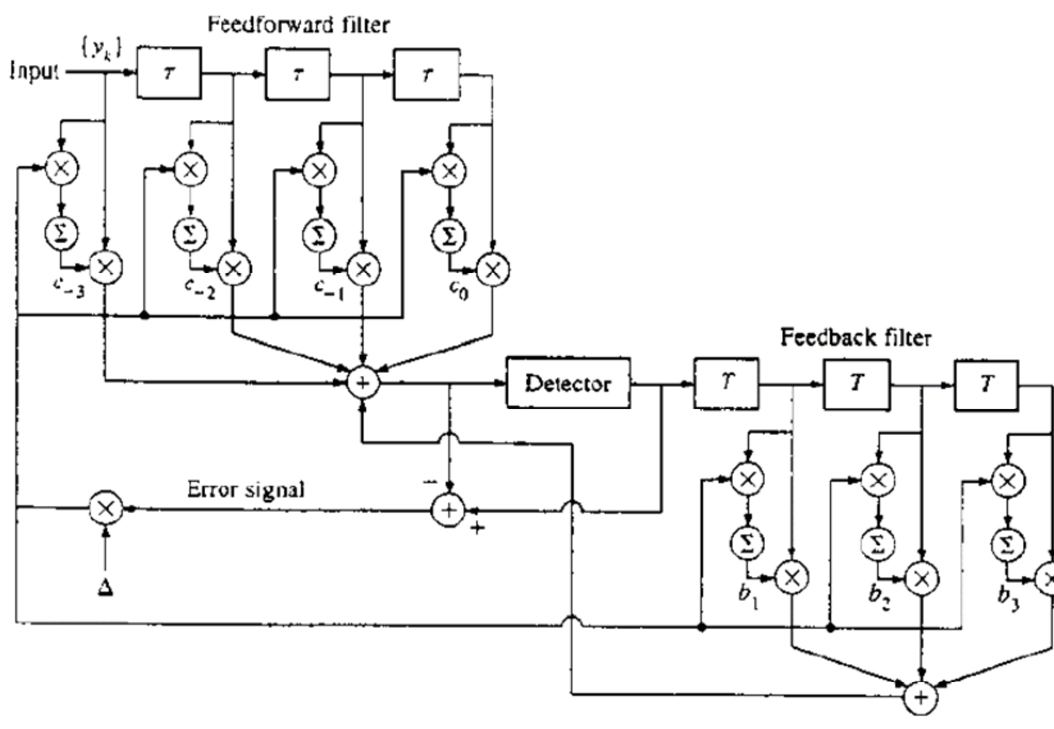


Figure 6.38: Adaptive DFE.

sequence $\{a_k\}$ from the received signal samples $\{y_k\}$ given in (6.4.6). In a digital communication system that transmits information over a channel that causes ISI, the optimum detector is a maximum-likelihood symbol sequence detector that produces at its output the most probable symbol sequence $\{\tilde{a}_k\}$ for the given received sampled sequence $\{y_k\}$. That is, the detector finds the sequence $\{\tilde{a}_k\}$ that maximizes the *likelihood function*

$$\Lambda(\{a_k\}) = \ln p(\{y_k\} \mid \{a_k\})$$

where $p(\{y_k\} \mid \{a_k\})$ is the joint probability of the received sequence $\{y_k\}$ conditioned on $\{a_k\}$. The sequence of symbols $\{\tilde{a}_k\}$ that maximizes this joint conditional probability is called *the maximum-likelihood sequence detector.*

An algorithm that implements maximum-likelihood sequence detection (MLSD) is the Viterbi algorithm, which was originally devised for decoding convolutional codes

The major drawback of MLSD for channels with ISI is the exponential behavior in computational complexity as a function of the span of the ISI. Consequently, MLSD is practical only for channels where the ISI spans only a few symbols and the ISI is severe, in the sense that it causes a severe degradation in the performance of a linear equalizer or a decision-feedback equalizer. For example, Figure 6.39 illustrates the error probability performance of the Viterbi algorithm for a binary PAM signal transmitted through channel B (see Figure 6.35). For purposes of comparison, we also illustrate the probability of error for a DFE. Both results were obtained by computer simulation. We observe that the performance of the ML sequence detector is about 4.5 dB better than that of the DFE at an error probability of 10^{-4}. Hence, this is one example where the ML sequence detector provides a significant performance gain on a channel with a relatively short ISI span.

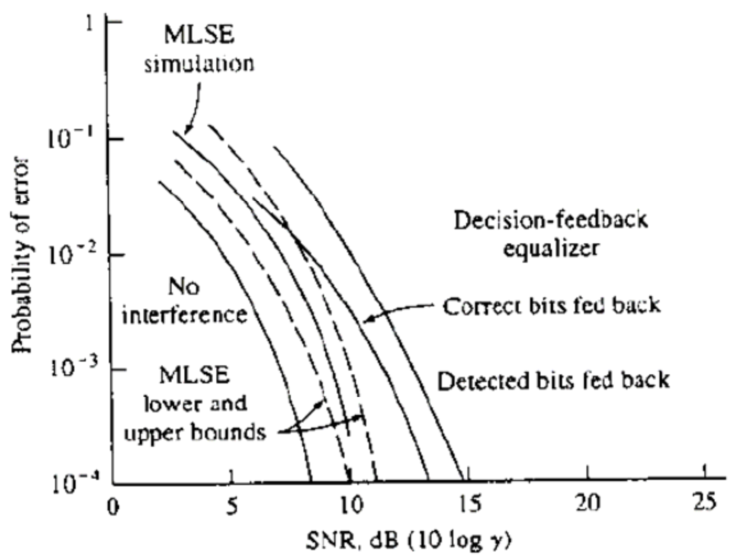


Figure 6.39: Error probability of the Viterbi algorithm for a binary PAM signal transmitted through Channel B in Figure 6.35.

In conclusion, channel equalizers are widely used in digital communication systems to mitigate the effects of ISI cause by channel distortion. Linear equalizers are generally used for high-speed modems that transmit data over telephone channels. For wireless (radio) transmission, such as mobile cellular communications and interoffice communications, the multipath propagation of the transmitted signal results in severe ISI. Such channels require more powerful equalizers to combat the severe ISI. The decision-feedback equalizer and the MLSD are two nonlinear channel equalizers that are suitable for radio channels with severe ISI.